On the Classical Foundations of Say’s Law: a Pasinetti Pure Labour Approach

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Say-Mill Hypothesis

‘one can make purchases only with what one has produced’ (Say 1803, pp. 175-180).

‘this demand, is exactly equal to the amount of what he has produced and does not mean to consume’; hence ‘the supply and demand of every individual are of necessity equal’ (Mill 1821, p. 189)

Say-Mill Hypothesis

$$p_i S_i = \sum_{j=1}^{m} p_j D_{ji} \qquad (1)$$

The Say-Mill Proposition. $p_i S_i = \sum_{j=1}^{m} p_j D_{ji}$ *implies* $\sum_{i=1}^{m} p_i S_i = \sum_{i=1,\, j=1}^{m} p_j D_{ji}$.

Proof. Aggregate equation (1) across m producers.

Deriving the Say-Mill Macroeconomic Condition

$$p_1 S_1 = p_1 D_{11} + p_2 D_{21} + \cdots + p_m D_{m1}$$

$$p_1 S_1 = p_1 \frac{D_{11}}{N_1} N_1 + p_2 \frac{D_{21}}{N_1} N_1 + \cdots + p_m \frac{D_{m1}}{N_1} N_1$$

$$p_1 = p_1 c_{11} l_1 + p_2 c_{21} l_1 + \cdots + p_m c_{m1} l_1$$

$$p_1 = w_1 l_1 \qquad \text{where } w_i = p_1 c_{1i} + p_2 c_{2i} + \cdots + p_m c_{mi}$$

Deriving the Say-Mill Macroeconomic Condition

$$
\begin{bmatrix} p_1 & p_2 & \cdots & p_m & w_1 & w_2 & \cdots & w_m \end{bmatrix}
\begin{bmatrix}
1 & 0 & \cdots & 0 & -c_{11} & -c_{12} & \cdots & -c_{1m} \\
0 & 1 & \cdots & 0 & -c_{21} & -c_{22} & \cdots & -c_{2m} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1 & -c_{m1} & -c_{m2} & \cdots & -c_{mm} \\
-l_1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\
0 & -l_2 & \cdots & 0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & -l_m & 0 & 0 & \cdots & 0
\end{bmatrix}
= \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}
$$

The Say-Mill Macroeconomic Condition

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1m} \\ c_{21} & c_{22} & \cdots & c_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mm} \end{bmatrix}, \text{ and } \mathbf{L} = \begin{bmatrix} l_1 & 0 & \cdots & 0 \\ 0 & l_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & l_m \end{bmatrix}$$

The condition, in coefficient form, for the Say-Mill Hypothesis

$$\det(\mathbf{I} - \mathbf{LC}) = 0$$

Absence of a General Glut

$$\sum_{i=1}^{m} p_i G_i = \sum_{i=1,\, j=1}^{m} p_i S_{ij} - \sum_{i=1,\, j=1}^{m} p_i D_{ij} \equiv 0$$

Partial Gluts

$$S_1 = 50\,,\; S_2 = 18 \text{ and } S_3 = 11$$

Table 1 *Ex ante* physical demands for commodities

	1	2	3	Total demand
1	25	10	15	50
2	8	10	3	21
3	3	2	4	9

Table 2 *Ex ante* money flows between producers

	1	2	3	Total receipts
1	25	10	15	50
2	16	20	6	42
3	9	6	12	27
Total outlays	50	36	33	119

Marx's critique of Mill

'As Mill says purchase is sale etc., demand is supply and supply is demand. But they also fall apart and can become independent of each other'

'At a given moment, the supply of all commodities can be greater than the demand for all commodities, since the demand for the general commodity, money, exchange-value, is greater than the demand for all particular commodities…'

(Marx, 1968, *Theories of Surplus Value*, part II, p. 505)

Table 5 Three Money Circuits

1st circuit

	1	2	3
1	25		
2	16	20	
3	9		12

2nd circuit

	1	2	3
1		4	3
2			6
3		12	

3rd circuit

	1	2	3
1		6	12
2			
3			

***Ex post* realised money flows**

	1	2	3	Total receipts
1	25	10	15	50
2	16	20	6	42
3	9	6	12	33
Total outlays	50	42	33	125

Barter

Assume

- Goods are traded *without* the exchange of money

Under barter let $p_i = l_i$

Hence under the Say-Mill Hypothesis (for two producers):

$$l_1 S_1 = l_1 D_{11} + l_2 D_{21}$$

$$l_2 S_2 = l_1 D_{12} + l_2 D_{22}$$

Total supply is equal to total supply (Say's identity), and

$N = l_1(D_{11} + D_{12}) + l_2(D_{21} + D_{22})$, or

$1 = l_1(D_{11} + D_{12})/N + l_2(D_{21} + D_{22})/N$, hence

$1 = l_1 c_1 + l_2 c_2$ (The Pasinetti macroeconomic condition)

Two macroeconomic conditions

$$\det(\mathbf{I} - \mathbf{LC}) = 0$$

$$(1 - \mathbf{lc}) = 0$$

Barter (under correctly anticipated demand)

Assume

- Goods are traded *without* the exchange of money
- For each individual the needs of fellow producers ‘are known to him’ (Marx 1968, p. 508).

For two producers

$$D_1 = c_1 N$$

$$D_2 = c_2 N$$

Now assume the producers correctly anticipate demand:

$$S_1 = c_1 N$$

$$S_2 = c_2 N$$

Since $N = l_1 Q_1 + l_2 Q_2$, we can get duality:

Pasinetti price system

$$\begin{bmatrix} p_1 & p_2 & \cdots & p_m & w \end{bmatrix} \begin{bmatrix} 1 & 0 & \cdots & 0 & -c_1 \\ 0 & 1 & \cdots & 0 & -c_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -c_m \\ -l_1 & -l_2 & \cdots & -l_m & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Pasinetti quantity system

$$\begin{bmatrix} 1 & 0 & \cdots & 0 & -c_1 \\ 0 & 1 & \cdots & 0 & -c_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -c_m \\ -l_1 & -l_2 & \cdots & -l_m & 1 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_m \\ N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$