## Final project report for eSTEeM project:

Use of STACK to generate formative assessment for Level 3 pure mathematics

Keywords: STACK, computer aided assessment, formative assessment, examination preparation, revision, past papers, self-efficacy

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## Executive summary

In a study of exam revision impact, Cross et al found that $83.9 \%$ of students rated access to a sample examination paper useful or very useful [Cross, Whitelock, and Mittelmeier, 2016]. Many mathematics modules have been running for a number of years and have a large number of available past examination papers. As a result, students studying mathematics at the OU have often developed revision strategies that assume access to a large number of past examination papers (together with solutions). On newer modules this resource does not exist, and students have cited lack of past papers as a reason for deferring or for not taking a new module. Mathematics examination-like questions and solutions are time-consuming and expensive to write and check; therefore, it is not easy to produce a large number of sample papers.

Many mathematics modules use a computer algebra system called STACK to generate online short practice questions (SPQs) with feedback and solutions. The STACK system can be programmed so that the feedback produced for incorrectly answered questions depends on the mistakes made by the individual. These questions are used to build quizzes (SPQ-quizzes). For this project we used STACK to produce sets of long examination-like practice questions (ELPQs) with worked solutions and tailored feedback for a relatively new module (M303). These long examination-like questions were then used to build a randomly generated quiz that resembled a past exam paper (ELPQ-quiz) so that students could generate multiple instances of an example examination-like quiz, with feedback and example solutions, for practice.

We used a mixed methods approach to our evaluation of this project, performing a quantitative analysis of the engagement with these questions by 660 students, sending a questionnaire to just over 200 students and carrying out a qualitative analysis of semi-structured interviews undertaken with 12 students.

A thematic analysis of the structured interviews showed lack of past papers as a strong theme, along with the existence of established revision strategies that relied on access to many past papers.

The results of the quantitative analysis showed that a higher percentage of students engaged with the ELPQ quiz than engaged with the traditional SPQ quizzes. The students that did engage with the ELPQs used them at a higher intensity. In addition, a significantly higher percentage of students who engaged well with the ELPQs maintained or improved their exam result for M303: Further pure mathematics, as compared to their score in the level two precursor module, M208: Pure mathematics.

Over $80 \%$ of students responding to the survey, taking the exam, and using the ELPQs, either agreed or strongly agreed that the ELPQ-quiz helped them structure their revision and felt that the ELPQs were helpful when they saw the exam. Over $90 \%$ of the respondents who used the ELPQs agreed or strongly agreed that quizzes similar to the ELPQ-quiz should be implemented on other mathematics modules.

## Aims and scope of the project

M303 is a relatively new 60 credit level 3 pure mathematics module and one of three Level 3 pure mathematics modules offered by the School of Mathematics and Statistics. The module text consists of six hard copy text books. There is a formative electronic `practice quiz' (SPQ-quiz) associated with each of the six books. These quizzes consist of varying numbers of short practice questions (SPQs).

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## Example of a practice quiz question and solution

```
In which of the following groups is the greatest order of an element 12?
Select one:
    \mp@subsup{\mathbb{Z}}{2}{}\times\mp@subsup{\mathbb{Z}}{4}{}
    \mp@subsup{\mathbb{Z}}{3}{}\times\mp@subsup{\mathbb{Z}}{18}{}
    Z}\mp@subsup{\mathbb{Z}}{2}{}\times\mp@subsup{D}{6}{
    \mp@subsup{\mathbb{Z}}{2}{}\times\mp@subsup{\mathbb{Z}}{2}{}\times\mp@subsup{\mathbb{Z}}{3}{}
    Z}\mp@subsup{\mathbb{Z}}{3}{}\times\mp@subsup{\mathbb{Z}}{4}{
```

Your answer is incorrect.
Cauchy's Theorem states that there is an element of each prime order dividing the order of a group (Cauchy's Theorem, Chapter 6, Theorem
3.16, Handbook page 34).
Given any direct product $G=A \times B$, the order of an element $(a, b) \in G$, is the lowest common multiple of the orders of $a$ and $b$,
respectively, in the groups $A$ and $B$ (Chapter 6, Proposition 2.8 , Handbook page 32).
Knowledge of small groups such as these provides an important source of counter-examples, especially the non-abelians ones formed from
the direct product of a cyclic group and a non-abelian group.
The correct answer is:
$\mathbb{Z}_{3} \times \mathbb{Z}_{4}$
The correct answer is: $\mathbb{Z}_{3} \times \mathbb{Z}_{4}$

Some of these SPQs (such as the one above) were created using the standard Moodle question types; but others were developed using STACK (the System for Teaching and Assessment using a Computer algebra Kernel), an online open-source software system that uses a computer algebra system (Maxima) to generate randomised instances of questions [Sangwin 2007].

For this project the module team used STACK to generate randomised examples of long, examinationtype questions and solutions (ELPQs) and built a quiz using these questions (the ELQP-quiz), so that the students could practise longer exam type questions and then mark their own attempts using the electronically generated specimen solutions.

Our research questions were:

- Will student engagement with the ELPQ-quiz be higher than engagement with the existing short question quizzes?
- Will the students who have access to the ELPQ-quiz feel better prepared for (and confident about) the exam?


## Background and motivation

By the time that they reach Level 3, students often have a tried-and-tested revision strategy with which they feel confident. For many mathematics students, their strategy assumes the existence of a large number of past papers with solutions. M303: Further pure mathematics follows on from a Level 2 module (M208: Pure mathematics) that has past papers available from 2006. The other pure mathematics Level three modules running alongside M303 have past papers available from 2002. No official solutions exist for these papers, but student-produced sets, and unofficial tutor-produced sets, are readily available on the internet.

The module team wanted to produce a substitute for past papers on M303. There were two reasons for this.

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Firstly, to provide practice at long examination-type questions for the students. In 2013, Dunlosky et al investigated ten different learning techniques [Dunlosky, et al, 2013] and found that "the advantage of practice testing with feedback over restudy is extremely robust" and that "Practice testing with feedback also consistently outperforms practice testing alone". However good feedback is necessary for there to be a pedagogic advantage [Gibbs and Simpson, 2005] and this feedback needs to be available promptly [Chickering and Gamson, 1987]. The existing short practice questions and quizzes on M303 were not suitable as a means of enabling students to practice completing longer, exam style questions. This is because these short questions were typically designed around simple answers (usually an equation, number or mathematical expression) that could be marked as right or wrong by a computer algebra system. Longer, exam-style questions required more complex answers, such as mathematical proofs or textual explanations that cannot be easily checked using the current computer system. Rather than using the computer system to check the answers to the ELPQs, we used the system to generate worked solutions that the students could then use to check their own work against the supplied answer. In addition, using STACK enabled the feedback and explanations to be based on the specific mistakes made by each student.

Secondly, we hoped to increase the confidence levels of our students by improving their mathematical self-efficacy, defined by Bandura as "people's judgments of their capabilities to organize and execute courses of action required to attain designated types of performances" [Bandura 1986]. As may be expected, personal experience has a strong influence on self-efficacy beliefs; with success raising it and failure lowering it [Bandura 1978]. Evidence suggests that knowledge learnt when taking practice tests can be flexibly applied to other contexts [Roediger and Butler, 2011] but it appears that this experience may need to be task-specific in order to develop positive mathematical self-efficacy [Borgonvi and Pokropek 2019]. Therefore, whilst the short question practice tests may increase students' ability to successfully complete examination questions, a good supply of longer examination like questions may be necessary to enable all students to get to the stage where they believe that they can successfully complete them.

Long questions (such as those found in examinations) with comprehensive sets of solutions are difficult and costly to produce for pure mathematics at this level and their availability is negatively affected by resource implications [Gibbs and Simpson, 2005]. However, students are becoming increasingly familiar with computer-aided assessment [Greenhow 2015] and so the module team decided to use STACK to create a set of examination-like practice questions that could be used to create a randomly generated set of practice examination-like papers.

## Activities

Over a period of four years, the module team continued to develop the ELPQs whilst analysing reaction by the students to the quizzes.

We decided upon a mixed methods approach to the analysis as defined by Johnson et al [Johnson, R. B., Onwuegbuzie, A. J., \& Turner, L. A. 2007] in order to expand our viewpoint [Greene, J., Caracelli, V., \& Graham, W. 1989].

For the 16 J presentation, the Staff Tutor on the module created a short survey containing questions about the ELPQs. Each of the twelve tutors was asked to email their tutor group, asking their students to consider filling in the survey. Of the 211 students remaining at the end of the module, 58 (27\%) filled in the survey.

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Each tutor was also invited to nominate students who may want to take part in semi-structured interviews. Interviews were carried out by one tutor, Joe Kyle, with twelve students.

Data showing all attempts at the SPQ-quizzes and the long ELPQ-quizzes for $15 \mathrm{~J}, 16 \mathrm{~J}, 17 \mathrm{~J}$ and 18 J were downloaded from the VLE. For each student the number of attempts at each quiz was logged and cross-referenced against that student's final exam scores, where available, on both M303 and M208 (the precursor module to M303).

Initially we only intended to analyse quantitative data from 15 J and 16 J . We changed this to include 17 J and 18 J because the project took longer than expected and the additional data became available.

The survey results and VLE data were analysed qualitatively and a thematic analysis carried out on transcripts of the structured interviews.

## Findings

Engagement and performance were both considered.

## Level of engagement with quizzes

The ELPQs were revision resources and only because visible to the students during the revision period at the end of the module. Therefore, only the 659 students who were still registered at the end of the module presentation were considered in the following analysis.

The ELPQ-quizzes were very popular with students. The following analysis compares engagement with the ELPQ-quizzes by the 659 students still engaged at the end of the module over four presentations, with engagement by the same group of students on the six SPQ-quizzes.

## Number of attempts at the quizzes.



[^0]| Presentation | PQ1 | PQ2 | PQ3 | PQ4 | PQ5 | PQ6 | ELPQ | Number <br> of <br> students |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 15J | 279 | 282 | 244 | 254 | 199 | 197 | 478 | 214 |
| 16J | 298 | 286 | 209 | 236 | 205 | 186 | 410 | 210 |
| 17J | 160 | 179 | 135 | 137 | 123 | 114 | 336 | 124 |
| 18J | 109 | 137 | 101 | 104 | 88 | 81 | 483 | 111 |

There were considerably more attempts made by students finishing the module on the ELPQ-quiz than there were on any of the SPQ-quizes. In addition a higher percentage of students engaged at least once with the ELPQ-quiz than with most of the SPQs.

Percentage of students engaging with each of the six SPQ-quizes at least once.


| Presentation | SPQ1 | SPQ2 | SPQ3 | SPQ4 | SPQ5 | SPQ6 | ELPQ | Number <br> of <br> students |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 15J | $72 \%$ | $71 \%$ | $65 \%$ | $68 \%$ | $63 \%$ | $60 \%$ | $78 \%$ | 214 |
| 16J | $81 \%$ | $70 \%$ | $58 \%$ | $65 \%$ | $59 \%$ | $61 \%$ | $74 \%$ | 210 |
| 17J | $81 \%$ | $82 \%$ | $70 \%$ | $70 \%$ | $71 \%$ | $64 \%$ | $87 \%$ | 124 |
| 18J | $68 \%$ | $76 \%$ | $67 \%$ | $69 \%$ | $64 \%$ | $56 \%$ | $84 \%$ | 111 |

One theme to emerge from the thematic analysis of the semi-structured interviews was the fact that the students believed that the authors of the exam were the same as the authors of the ELPQ. For example, one student said "I'm also thinking - hang on, someone's made those quizzes up. The chances are the people make these up have something to do with writing the exam". It may be that engagement with the exam like quizzes was higher for this reason.

Of the students answering the survey, $74 \%$ agreed or strongly agreed that the ELPQs helped them to structure their revision and $76 \%$ agreed or strongly agreed that the ELPQs were generally helpful.

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## Performance of students

The score on M208, the precursor module, was used to establish a baseline value for each student in the following. Therefore, only the 549 students who had available scores for both M303 and M208 were considered for this part of the analysis.

## Grouping students by quiz engagement

There is a positive correlation (0.55) between M208 exam score and quiz engagement

|  |  | Mean number <br> Mumber of <br> students | Mean number <br> attempts per <br> student | Mean number <br> of SPQ-quiz <br> attempts per <br> student | Percentage of <br> attempted by <br> each student* |
| :--- | :--- | :--- | :--- | :--- | :--- |
| students with <br> no attempts <br> at ELPQ-quiz |  |  |  |  |  |
| $85 \%$ and over | 227 | 3.5 | 9.8 | 5.2 | 4.0 |
| $55 \%$ to $84 \%$ | 240 | 2.6 | 6.1 | 4.1 | 15.8 |
| $54 \%$ or less | 103 | 2.2 | 4.0 | 3.3 | 23.2 |

* There were six available SPQ quizzes that could be attempted.

Therefore the students were divided into three groups: 227 students who scored $85 \%$ or over on M208 (these roughly equate to students with a grade 1 pass on M 208 ); 240 students who scored between $55 \%$ and $84 \%$ on M208 (these roughly equate to students with a grade 2 and grade 3 pass on M 208 ) and 82 students who scored less than $55 \%$ on M208 (these roughly equate to students with a grade 4 pass or a fail on M208).

Looking at these groups separately enabled a comparison to be made between the M303 scores of groups of students who had very similar average M208 scores but different quiz behaviour profiles.

## Students with 85\% or over on M208

| Number of <br> quiz <br> attempts | Number of <br> students | Mean <br> M208 score | Mean <br> M303 score |
| :--- | :--- | :--- | :--- |
| $\mathbf{3}$ or more | 128 | $93.6 \%$ | $84.4 \%$ |
| $\mathbf{2}$ | 34 | $92.9 \%$ | $83.2 \%$ |
| $\mathbf{1}$ or fewer | 65 | $92.9 \%$ | $81.5 \%$ |

Students scoring more than $85 \%$ on M208 who did the test three or more times appear to have higher scores than those who did it two or fewer times (z-test score 1.48, p<0.05).

[^1]
## Students scoring between 55\% and 84\% on M208

| Number of <br> quiz <br> attempts | Number of <br> students | Mean <br> M208 score | Mean <br> M303 score |
| :--- | :--- | :--- | :--- |
| $\mathbf{3}$ or more | 85 | $71.9 \%$ | $70.2 \%$ |
| $\mathbf{2}$ | 33 | $71.8 \%$ | $68.5 \%$ |
| $\mathbf{1}$ or fewer | 122 | $70.2 \%$ | $57.0 \%$ |

The scores of students scoring between $55 \%$ and $84 \%$ on M208 who have taken the ELPQ two or more times appear higher than those how have taken it one or fewer times (z-test score $7.00, \mathrm{p}<0.01$ ).

## Students scoring less than 55\% on M208

| Number of <br> quiz <br> attempts | Number of <br> students | Mean <br> M208 score | Mean <br> M303 score |
| :--- | :--- | :--- | :--- |
| 3 or more | 25 | $45.8 \%$ | $47.1 \%$ |
| $\mathbf{2}$ | 9 | $46.7 \%$ | $42.7 \%$ |
| $\mathbf{1}$ or fewer | 48 | $45.1 \%$ | $42.2 \%$ |

Students scoring below $55 \%$ on M208 who did the test three or more times appeared to have higher scores than those who did it two or fewer times (z-test score 1.36).

The z-test score and the difference in mean scores for students scoring between $55 \%$ and $84 \%$ on M208 are particularly significant.

## Grouping students by exam performance

The difference between an individual's examination score on M303 and score on M208 (as a percentage of M208 score) was used in order to measure performance on M303 whilst taking into account the student's initial mathematical background knowledge.

There was a small positive correlation between the difference in score between M303 and M208 scores (as a percentage of M208 score) and the number of quiz attempts.

Correlation between score differences and ELPQ attempts: 0.189 .
Correlation between score differences and other PQ attempts: 0.051 .
Students were divided into two groups: those who had an M208 score that was either higher than their M208 score or within 10\% of their M208 score and students who scored more than $10 \%$ less on M303 than on M208.

[^2]|  | Number of <br> students | Average <br> M208 score | Average <br> attempts at <br> ELPQ-quiz | Average <br> attempts at <br> SPQ-quizzes |
| :--- | :--- | :--- | :--- | :--- |
| M303 score 90\% or more of <br> M208 score | 295 | 76.8 | 3.5 | 7.9 |
| M303 score less than 90\% of <br> M208 | 254 | 76.0 | 2.3 | 6.6 |

The numbers of attempts at the ELPQ-quiz made by students who's M303 score was more than $90 \%$ of their M208 score appeared higher than the numbers of attempts at the ELPQ-quiz made by students who's M303 score was more than $90 \%$ of their M208 score based on a z-test value of 4.02 ( $p<0.01$ ). However, a z-test value of 1.23 showed no significant difference between the numbers of attempts at the SPQ-quizzes between these two groups.

## Confidence of students

When asked to think about how they felt before seeing the exam, the survey respondents replied as follows.


A thematic analysis [Braun and Clarke, 2006] of the transcripts of semi-structured interviews with 12 students was carried out. A number of themes emerged including:

- Mimicking the examination experience: when talking about what they actually did when attempting the ELPQs students appeared to be trying to create a realistic examination experience. When speaking of their solutions comments include "I would write them out as if I was writing out a full answer to an exam question"; "I felt that I needed to write them out because I thought of myself as practising for the exam. That's all part of it, isn't it?"
- Preparedness: this was a strong theme with several subthemes.
- Spotting gaps: students spoke of using the ELPQs to check how prepared they were. "I did want to tackle them as if I had just been presented with them in an exam and then see how well I would do"; "the quizzes were essential to providing a check on 'do I know this'"; "[the ELPQs] helped me see very quickly where my gaps were."
- Confirming preparedness: students were also using them to confirm that they were indeed prepared. "I would use them to check before I moved forward."

[^3]- Gratitude and enjoyment: students did not view the ELPQs as part of the `standard offerings' but as an extra bonus, for which they were thankful. "It must have taken some time to get them up there and I'm really grateful for that effort"; "I'm very grateful they were there, basically"; "I just think it was like a brilliant bonus". "I really enjoyed the answers" This indicates that the students viewed the experience of using them positively and suggests that, for these students at least, the questions made them feel good by confirming that they were prepared for the exam (which would increase confidence levels) rather than highlighting a lack of preparedness.


## Summary

The above findings suggest that students may be more likely to engage with formative assessment in the form of online practice questions if they believe them to be directly relevant to the exam. Encouraging engagement is important because the results suggest that active engagement with online practice questions may help performance at pure mathematics at Level 3, especially for students with grade 2 or 3 passes in relevant modules at Level 2 . Students can feel more confident if given plenty of opportunity to self-test on resources specifically designed to mimic examinations.

## Impact

These results will be disseminated and will inform the development of other similar quizzes. In addition, similar techniques could possibly be used to create TMAs, individualised TMAs or TMA-like questions.

## List of deliverables

Intended publication.

[^4]
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[^5]
## Appendices

## Example of short practice quiz question and solution (using STACK)

## You should be able to attempt this question after you have read Chapter 2.

This question uses a computer algebra system to check your answer and it is important to write your answer in a way that the computer will understand: it expects you to enter a positive integer in digits, without any commas or other punctuation. How many distinct factors does the number $3240125=5^{3} \times 7^{2} \times 23^{2}$ have?
The number 3240125 has 7 distinct factors.

Incorrect answer.
The correct answer is 36 .

The number of distinct factors is given by the $\tau$ function (Definition 2.2 Chapter 2, HB page 17). The formula for the $\tau$ function is $\tau(n)=\left(k_{1}+1\right) \times \cdots \times\left(k_{r}+1\right)$ where $n=p_{1}^{k_{1}} \cdots p_{r}^{k_{r}}$ (see Proposition 2.3 of Chapter 2, HB page 17).

Since $3240125=5^{3} \times 7^{2} \times 23^{2}$, the number of distinct factors of 3240125 is

$$
(3+1)(2+1)(2+1)=36
$$

See Book A Chapter 2 Page 82.

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## Example of long exam-like question

Exam Practice, Book C, six marks
Q3(a)

This question uses a computer algebra system to check your answer and it is important to write your answer in a way that the computer will understand: it expects you to enter an integer

You should write out a justification for your answers before checking them
(i) Calculate $(1862 / 61)$ (note that $1862=49 \times 19 \times 2$ )
(1862/61) is $\square$
(ii) Calculate $(384 / 47)$ (note that $384=64 \times 2 \times 3$ )
$(384 / 47)$ is $\square$

## Check

Exam Practice, Book C, five marks

Q3(b)

Select the correct answer from the drop down list. You should write out a justification for your solution before checking it.

It is (No answer given) $v$ that Eisenstein's Criterion shows that the polynomial $f(x)=5 x^{4}+10 x^{3}+55 x^{2}-1 x+5$ is irreducible over $\mathbb{Q}[x]$

## Solution

3(a)
(i) $(1862 / 61)$ is -1 .
(ii) $(384 / 47)$ is 1

## Justification

(i) We have:
$(1862 / 61)=(49 / 61) \times(19 / 61) \times(2 / 61)$
since the Jacobi symbol is multiplicative by Definition 4.8 and Theorem 2.2 of Chapter 10 (HB pages 47 and 46).

Now $(49 / 61)=\left(7^{2} / 61\right)=1$ by Theorem 2.2 of Chapter 10 (HB page 46 ).
In addition, since $61 \equiv 5(\bmod 8)$, the result on the quadratic character of $2(\mathrm{HB}$ page 46$)$ gives $(2 / 61)=-1$.

Therefore
$(1862 / 61)=1 \times(19 / 61) \times-1$.

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So we need to calculate (19/61). The Law of Quadratic Reciprocity (alternative formulation - Theorem 4.2 of
Chapter 10, HB page 46) tells us that
$(19 / 61)=1 \times(61 / 19)$ (since both 19 and 61 are odd primes with $61 \equiv 1(\bmod 4)$ and $19 \equiv 3(\bmod 4)$ )

Therefore, since $61 \equiv 4(\bmod 19)$ we have $(19 / 61)=1 \times(61 / 19)=1 \times(4 / 19)=1$ by Theorem 2.2 (a) and (b) (HB page 46).

Therefore
$(1862 / 61)=1 \times 1 \times 1 \times-1$

$$
=-1
$$

(ii) Since $384=64 \times 2 \times 3$ and $64=8^{2}$ is a square number, we have

$$
\begin{aligned}
(384 / 47) & =1 \times(2 / 47) \times(3 / 47) \\
& =1 \times 1 \times 1 \\
& =1
\end{aligned}
$$

by Theorem 2.2, Proposition 3.2 and Proposition 4.7of Chapter 10 (HB page 46).

## 3(b)

Eisenstein's Criterion (Theorem 4.15 of Chapter 11 , HB page 52 ) tells us that the polynomial $f(x) \in \mathbb{Z}[x]$, is irreducible over $\mathbb{Q}[x]$ if we can find a prime, $p$, such that:
(i) $\boldsymbol{p}$ divides the constant term and the coefficients of $x, x^{2}$ and $x^{3}$, but does not divide the coefficient of $x^{4}$, and
(ii) $p^{2}$ does not divide the constant term.

Looking at $f(x)$ we see that the coefficient of $x^{2}$ is $55=11 \times 5$. Therefore our only choices for the prime $p$ are 5 or 11. Now the prime $p$ must also divide the constant term, 5 , but 11 clearly does not divide 5 . Therefore the only possibility is $p=5$

We check the conditions for each coefficient in turn.

- The prime 5 divides the constant term 5 , and the square 25 does not divide 5 , so this satisfies the conditions for Eisenstein's criterion.
- The prime 5 does not divide -1 , the coefficient of $x$, so this does not satisfy the conditions for Eisenstein's criterion.
- By design, the prime 5 divides the coefficient of $x^{2}$, so this satisfies the conditions for Eisenstein's criterion.
- The prime 5 divides 10 , the coefficient of $x^{3}$, so this satisfies the conditions for Eisenstein's criterion.
- The prime 5 divides 5 , the coefficient of $x^{4}$, so this does not satisfy the conditions for Eisenstein's criterion.

Therefore Eisenstein's Criterion cannot be used to deduce that $f(x)$ is irreducible.
Note that we do not need to check all of the conditions
to deduce that Eisenstein's criterion cannot be used here,
it is sufficient to find one case where the conditions are not met.

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## Semi structured interview questions

## Introduction

First some short background questions

1. Were you doing any modules alongside M303? If so which ones?
2. Did you use the exam-like practice quizzes?
3. Did you use the other practice quizzes?

## Practice quizzes - motivation

4. Why did you look at the ELPQs?

- (If they say that this was because the MT mentioned them - Were they as you expected? )
How did they use them

5. As a rule, did you print the questions? Or were you usually happy to work with the questions online?
6. Did you write out solutions?

- (If not then did you think through the solution before checking the answer, or did you just read it?)

7. Did you find that you could get the correct answer even if you didn't understand the theory?
8. Did you feel that you learnt anything new from the ELPQs?

## Effect of ELPQs on Exam experience

9. Did attempting the ELPQs make you feel better prepared for the exam?

- And when you saw the exam did this perception change?

Thoughts about the ELPQs
10. Did the quizzes help you structure your revision?

- If so, in what ways? If not, can you say why?

11. What was the most helpful thing about the ELPQs for you?
12. Did the quizzes help you to familiarise yourself with the `question types' that appeared in the exam?
13. For you, how might the ELPQs be improved?
14. Please give me any other thoughts you have about the ELPQs.

We now have a few final questions about how you approach learning mathematics, thinking particularly about M303.
15. Did you usually look at the TMA before or after reading the text books?
16. The study planner on M303 lists some parts as 'non-essential'. Did you know this?

- If yes then - did you read the non-essential parts of M303?

17. How do you usually approach revision? (sub-text - do they usually rely on past papers.)

Is there anything else that you would like to add?

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