

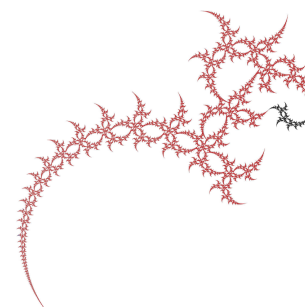
## 2024 PhD Projects

<b>Project title</b>	Limit sets of semigroups of hyperbolic isometries
<b>Principal supervisor</b>	Ian Short
<b>Second supervisor</b>	Vasiliki Evdoridou
<b>Discipline</b>	Pure mathematics
<b>Research area/keywords</b>	conformal dynamics, hyperbolic geometry, iterated function systems
<b>Suitable for</b>	Full-time applicants, Part-time applicants

### Project background and description

This project is about exploring the relationship between semigroups generated by (complex) Möbius transformations, iterated function systems associated to these semigroups, and the limit sets of the semigroups.

Associated to any Kleinian group is a subset of the extended complex plane called a limit set. For a semigroup of Möbius transformations, there are both forward and backward limit sets, and their interaction can inform us about the structure of the semigroup. The forward and backward limit sets of a semigroup generated by two parabolic Möbius transformations are shown in the figure; they intersect in two points, which correspond to the fixed points of the two parabolic maps.



Semigroups and limit sets of real Möbius transformations were explored in [2]. There it was found, for example, that if the forward and backward limit sets of a (real) semigroup of Möbius transformations coincide, then the semigroup is in fact a group. One challenge of this project is to generalise that result to complex semigroups of Möbius transformations, using three-dimensional hyperbolic geometry. Another challenge is to address some of the open problems from [1].

The project will begin with background reading, looking at texts such as Keen and Lalic's *Hyperbolic geometry from a local viewpoint*, Abate's *Holomorphic dynamics on hyperbolic Riemann surfaces*, and Marden's *Hyperbolic manifolds*. There will also be a starter research problem to get going with alongside background reading. The project will involve programming to generate mathematical artwork. The research student will meet the principal supervisor weekly, and they will also be invited to weekly meetings of the Geometry and Dynamics Group. Additionally, they will be able to attend the regular Dynamical Systems Seminars and other seminars.

### Background reading/references

- [1] Avila, Artur, Bochi, Jairo, and Yoccoz, Jean-Christophe, Uniformly hyperbolic finite-valued  $SL(2, \mathbb{R})$ -cocycles, *Comment. Math. Helv.* **85** (2010).
- [2] Jacques, Matthew and Short, Ian, Semigroups of isometries of the hyperbolic plane, *Int. Math. Res. Notices* **2022** (2021).