Project title: Lacunarity, Anisotropy and Shape Statistics of Dynamical Fractals

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Discipline: Applied mathematics
Research area/keywords: Fractals, Strange attractors, Scattering
Suitable for: Either full time or part time applicants

Project background and description
Fractal sets [1] appear naturally in models for many physical processes: examples include the distributions of galaxies in the cosmos, the distributions of stars within galaxies [2], the distribution of debris floating on the surface of the ocean [3], and the distribution of small particles, such as water droplets in clouds, in a turbulent flow. The underlying models are typically dynamical systems which have chaotic motion for which the measure converges to a strange attractor [4]. The fractal structure is usually characterised by considering the mass inside a small ball, which satisfies a power law relation with an exponent which is known as the fractal dimension [1]. This project is concerned with developing novel approaches to characterising the structure of physically important fractal sets, going beyond defining fractal dimensions. One aspect concerns the distribution of matter in regions of very low density. There is recent evidence that regions of very low density have a power-law distribution of density, which is distinct from the power-law distribution of high densities which defines the fractal dimension. Part of the project will investigate a ‘lacunarity exponent’, which characterises the sparsely occupied regions. Another aspect is that recent work has shown that the distribution of matter can can become increasingly anisotropic when examined on finer scales [5]. The project will also address how the statistics of the shape of ‘constellations’ of points in the fractal may become increasingly anisotropic when examined on an increasingly fine scale [6]. The physical implications for scattering of light from fractal distributions of matter will also be investigated, and possible applications in remote sensing and astrophysical imaging will be assessed.

Background reading/references