



**APERIODIC**

**TILINGS**



LONDON  
MATHEMATICAL  
SOCIETY  
EST. 1865

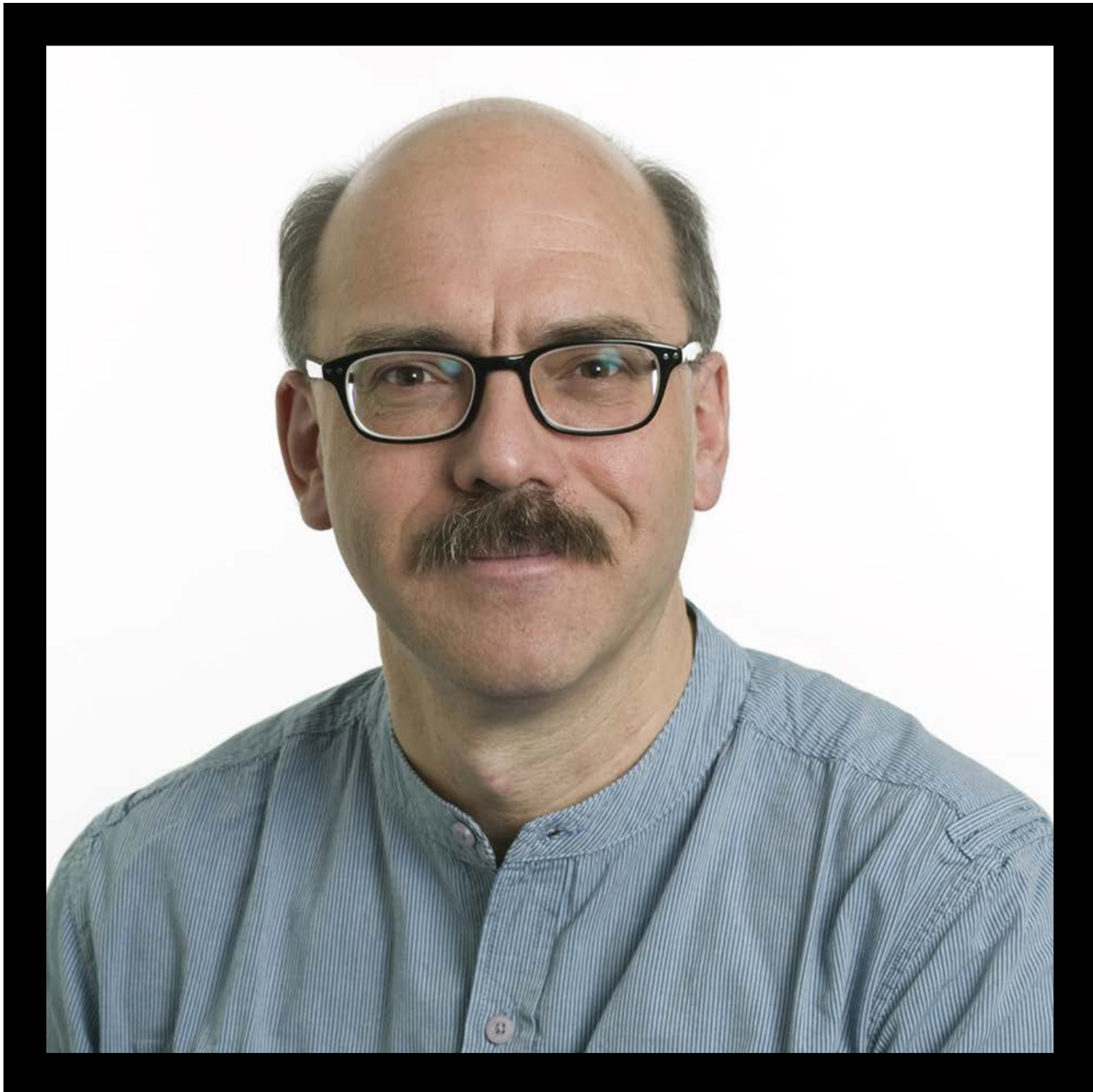


UNIVERSITÄT  
BIELEFELD





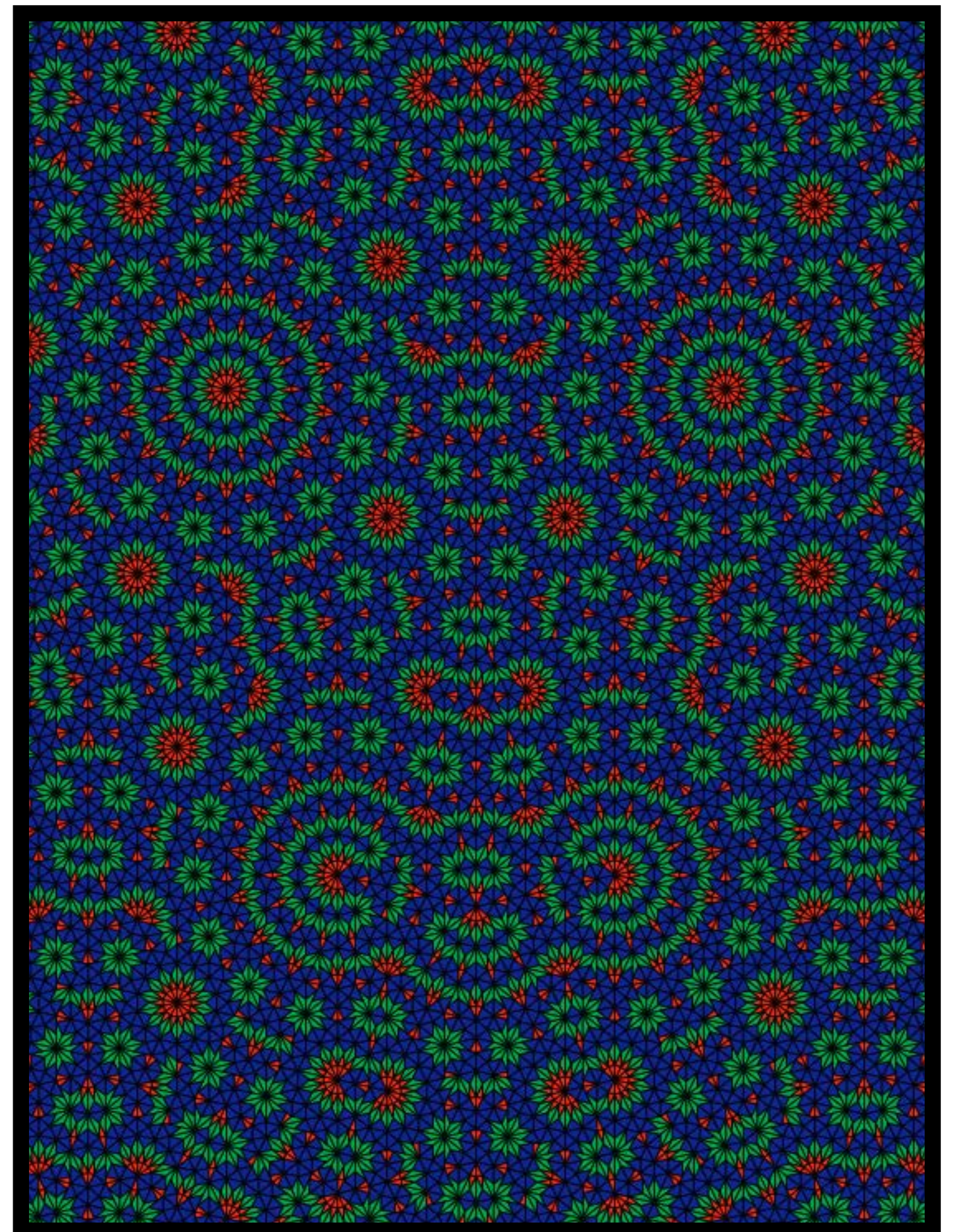
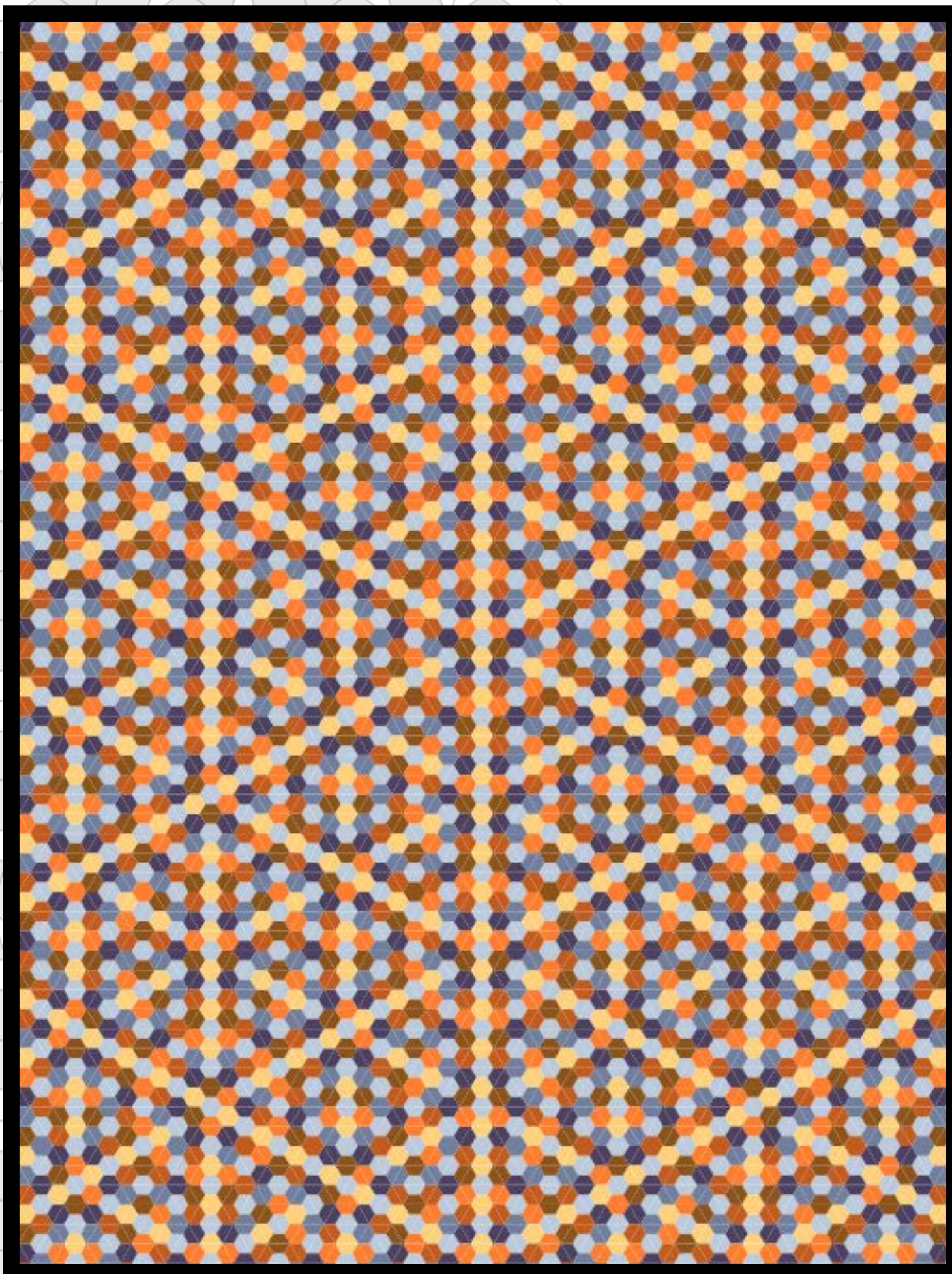
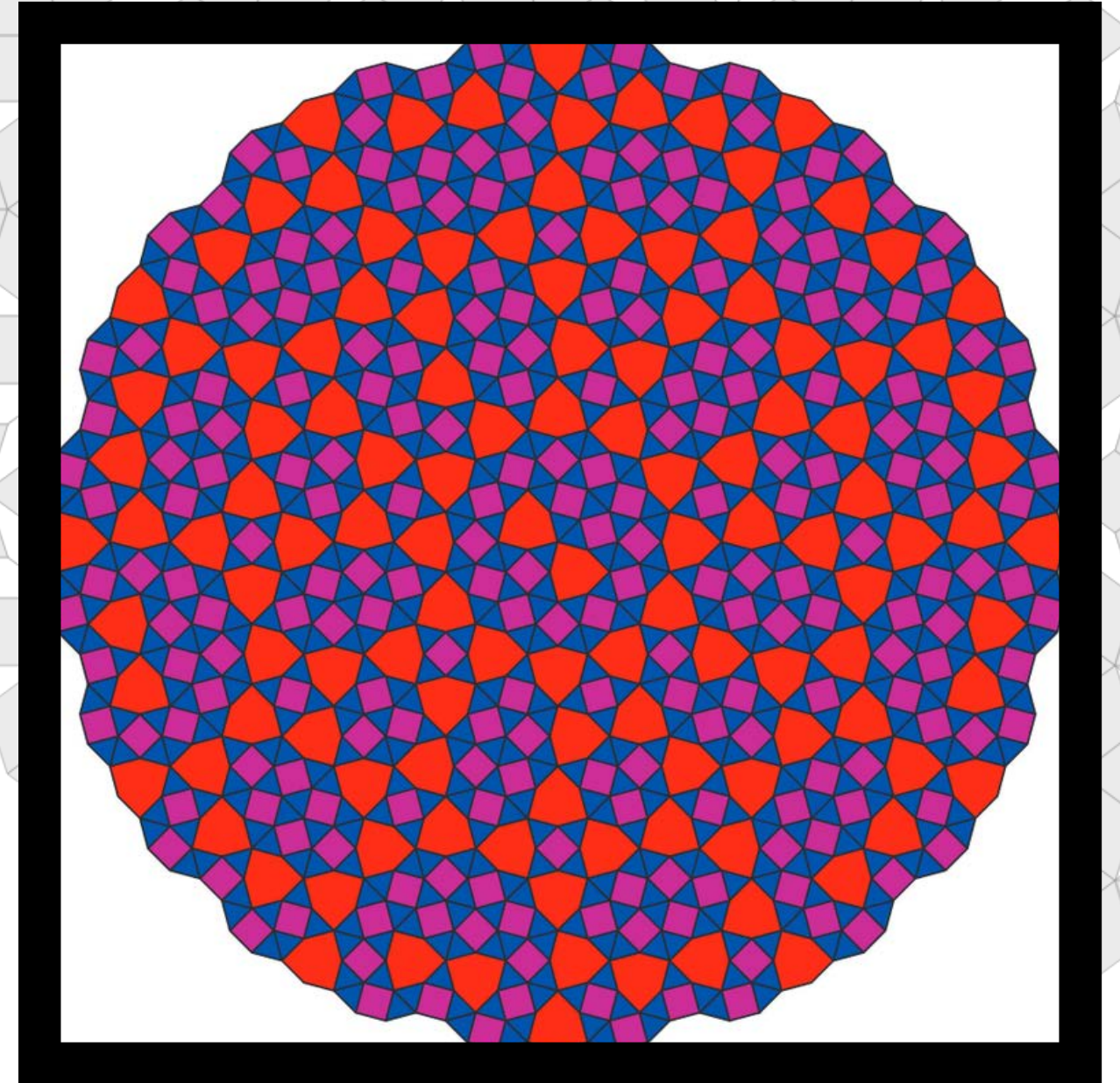
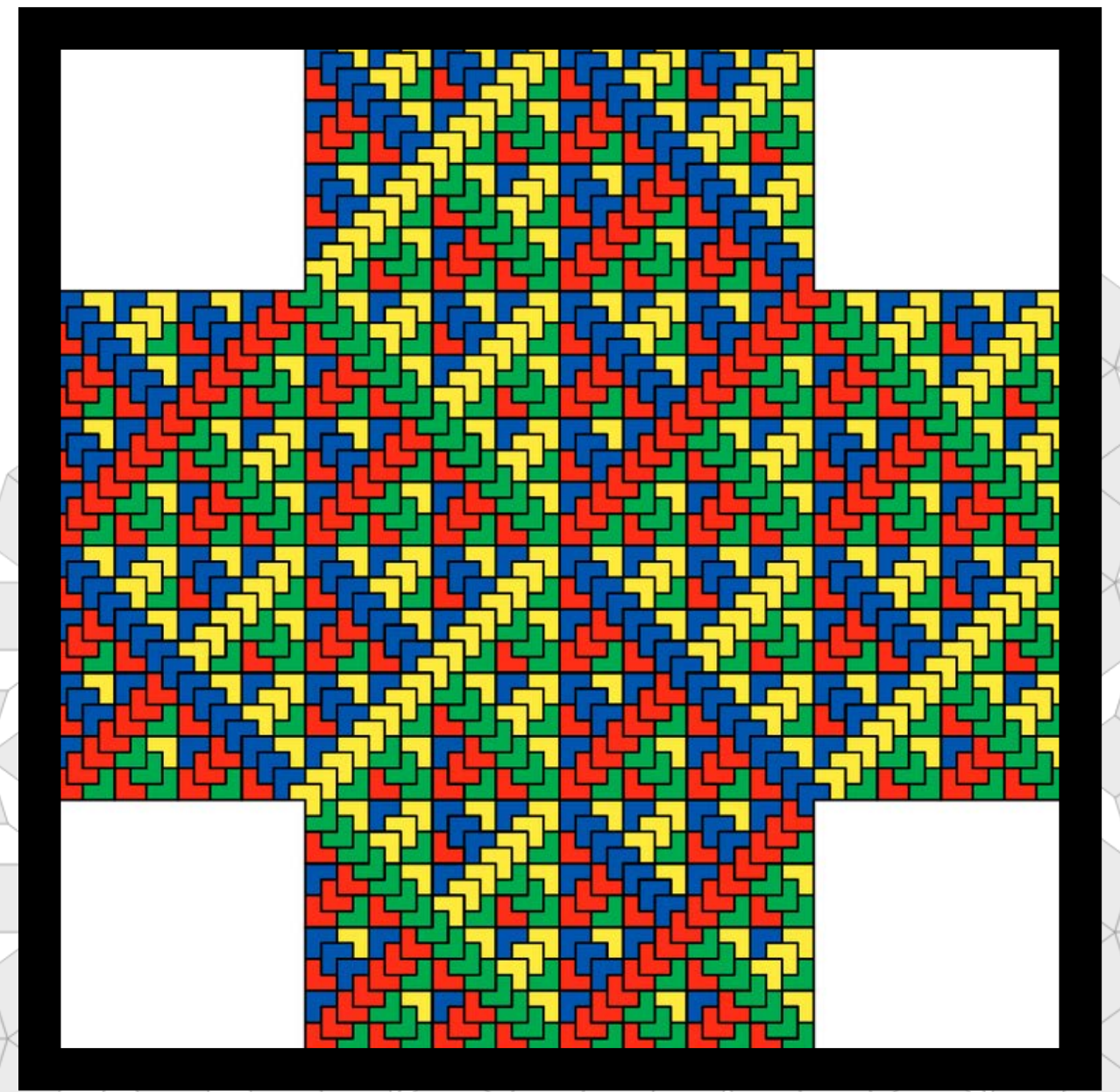
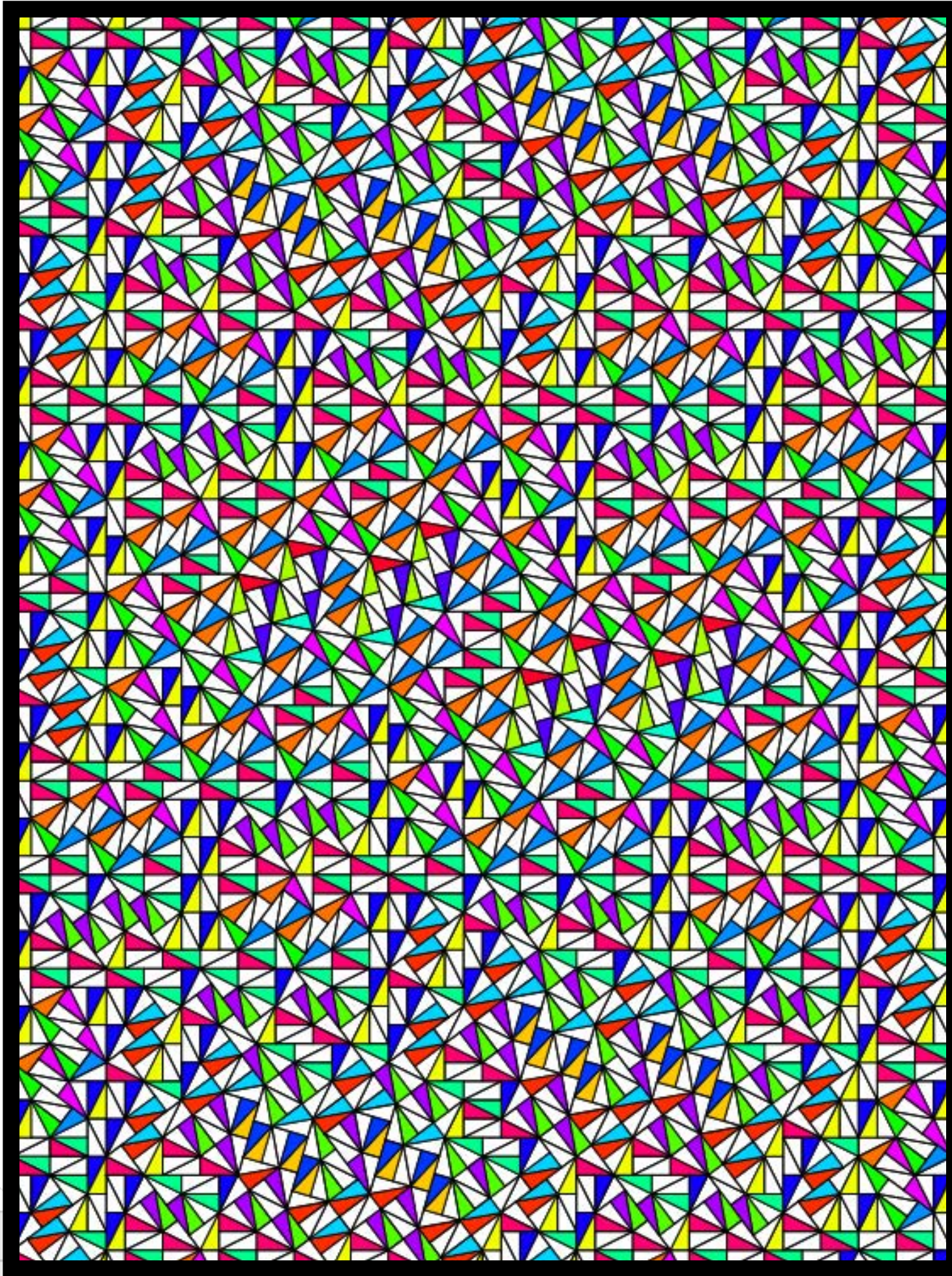
# IN MEMORY OF UWE GRIMM



Uwe's research career began in mathematical physics and later moved into mathematical biology, combinatorics, and the theory of aperiodic order, which is the mathematical study of the artwork displayed in this exhibition. He was a world leader in aperiodic tilings. Uwe and co-author Michael Baake wrote and edited the beautifully illustrated series of mathematical books *Aperiodic Order*, which are an invaluable resource for researchers in the field. Some of the images from the first text in this series are displayed at the exhibition. These same images were used by Uwe to convey the beauty of mathematics to school students and the public in many outreach events.



**These pieces were  
produced by Uwe Grimm  
with support from his  
friend Michael Baake**





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# CLARISSA GRANDI

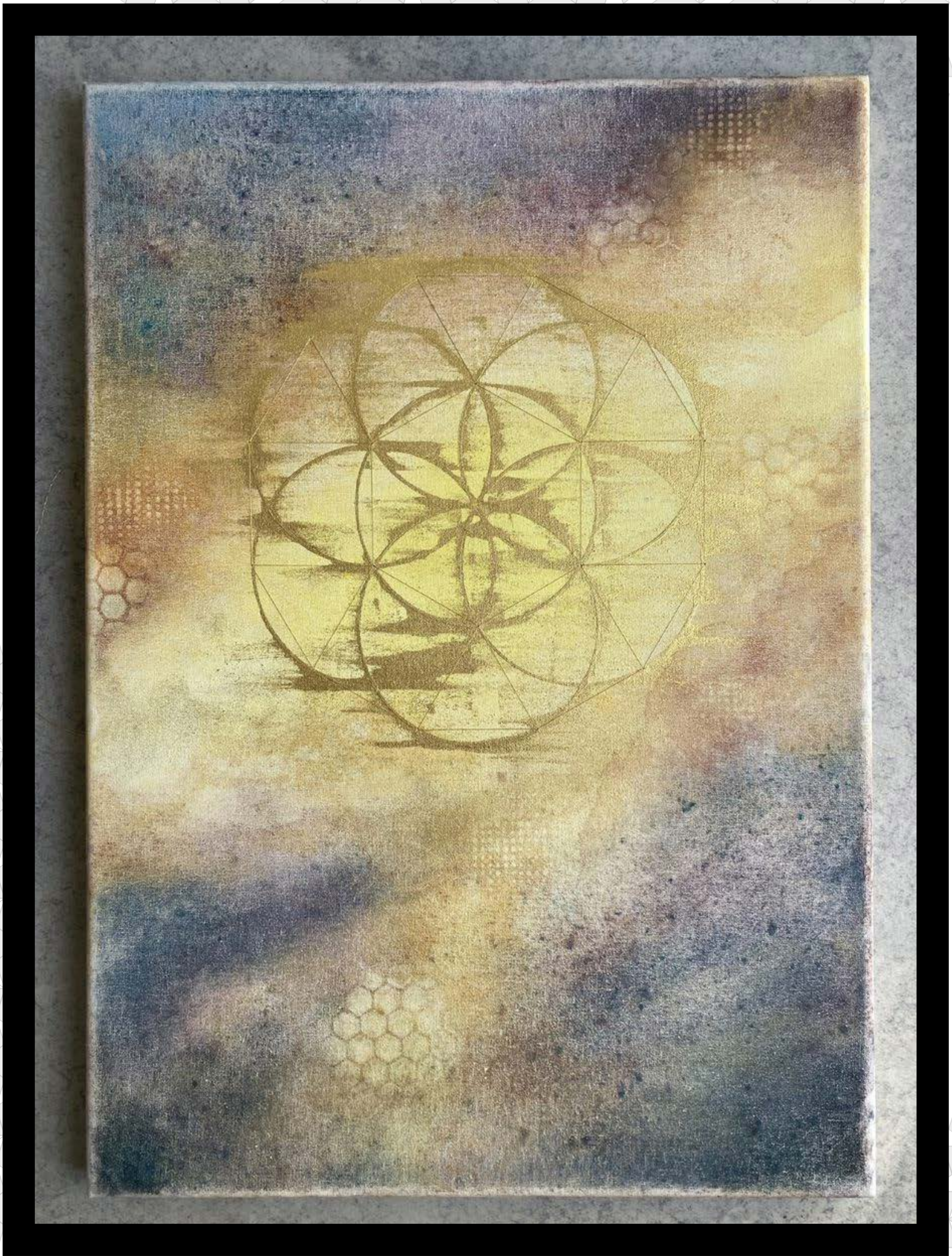
Clarissa teaches Post-16 Mathematics in a sixth form college in Suffolk, UK. She is the creator of [www.artfulmaths.com](http://www.artfulmaths.com), a mathematical art resources website and blog, and author of the Artful Math Teacher Book and Activity Book, published by Tarquin Publications. She also makes time to draw and paint geometric artwork, where she enjoys exploring the interplay between rigid, regular, human-made geometry and the more organic and chaotic geometries and symmetries found in the natural world. She has recently begun sharing her knowledge and techniques in a series of online geometric art courses for adults. Clarissa posts about all things maths and art on Twitter as [@c0mplexnumber](https://twitter.com/c0mplexnumber), and shares her artwork on Instagram as [@clarissagrandi.art](https://www.instagram.com/clarissagrandi.art).



To enquire about purchasing an artwork, please contact [clarissagrandi.art@gmail.com](mailto:clarissagrandi.art@gmail.com)







**Clarissa Grandi**



# EDMUND HARRISS

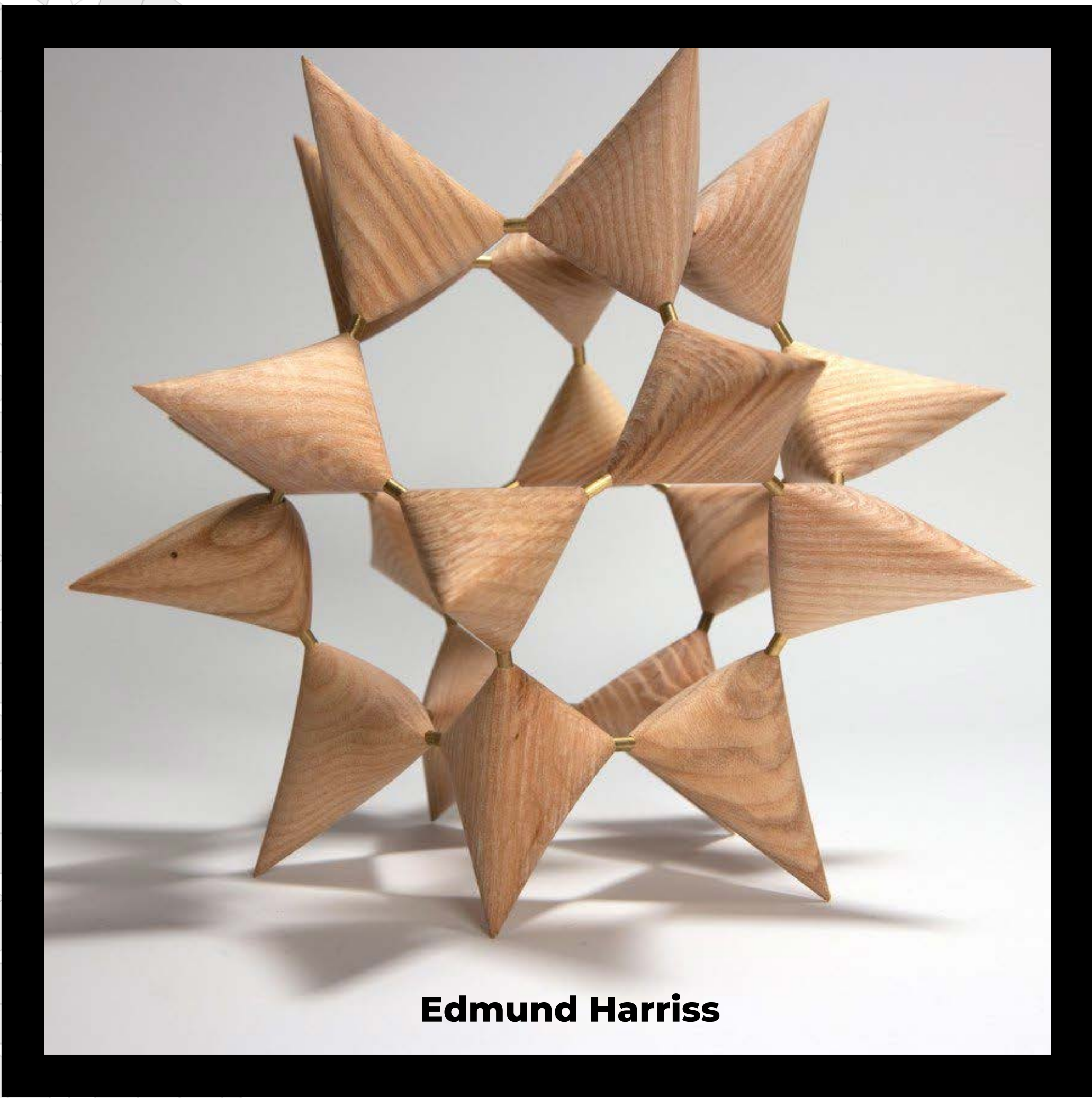
Edmund Harriss is a mathematician, teacher, artist and maker, in the Department of Mathematical Sciences at the University of Arkansas. His research veers from illustrating algebraic numbers, through the differential geometry of controlling CNC machines to mathematical art and perceptualism. His research has appeared in journals including Nature, and the proceedings of the National Academy of Science, as well as in the national and international media, including New Scientist, NPR, the Guardian, and Numberphile. His artwork is installed in several universities from Imperial College in London to the University of Arkansas, including a 12' metal sculpture currently (Fall 2020) being installed at the University of Arkansas. He has created two adult colouring books of mathematical images which open up a large range of mathematical topics to a wide range of people.



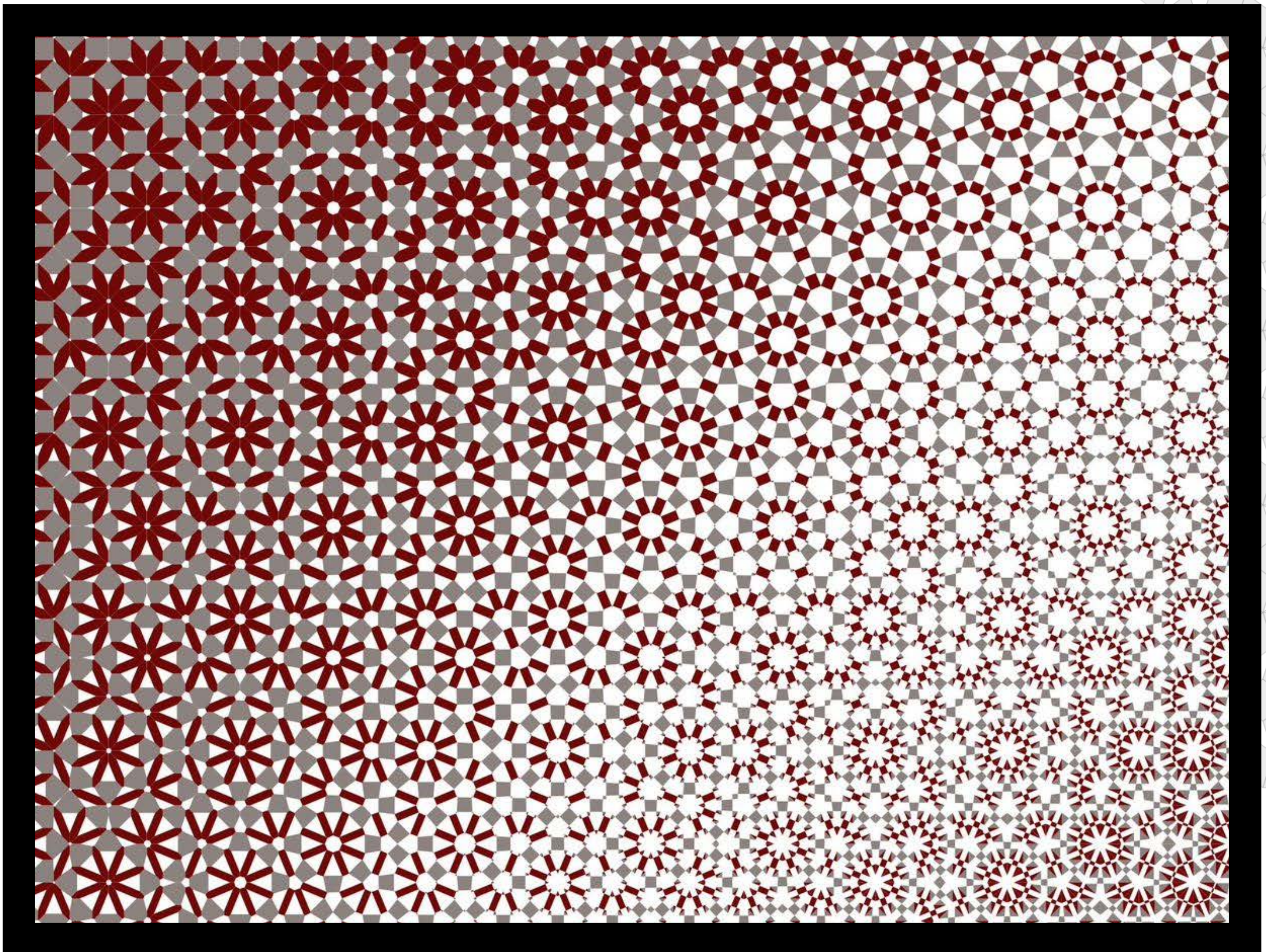
maxwelldemon.com  
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twitter: @Gelada







**Edmund Harriss**



**Edmund Harriss**



# HASRET BROWN

Hasret Brown is a visual artist living and working in Birmingham. She was born and raised in the Netherlands spending her teenage years and early twenties in Turkey where she absorbed the art, culture and traditions that have been a crucial element to understanding the roots of her identity and influencing the direction of her artwork. She specialises in Islamic geometric abstract art, taking influences from her own heritage and cultural background as well as across all Islamic Lands.

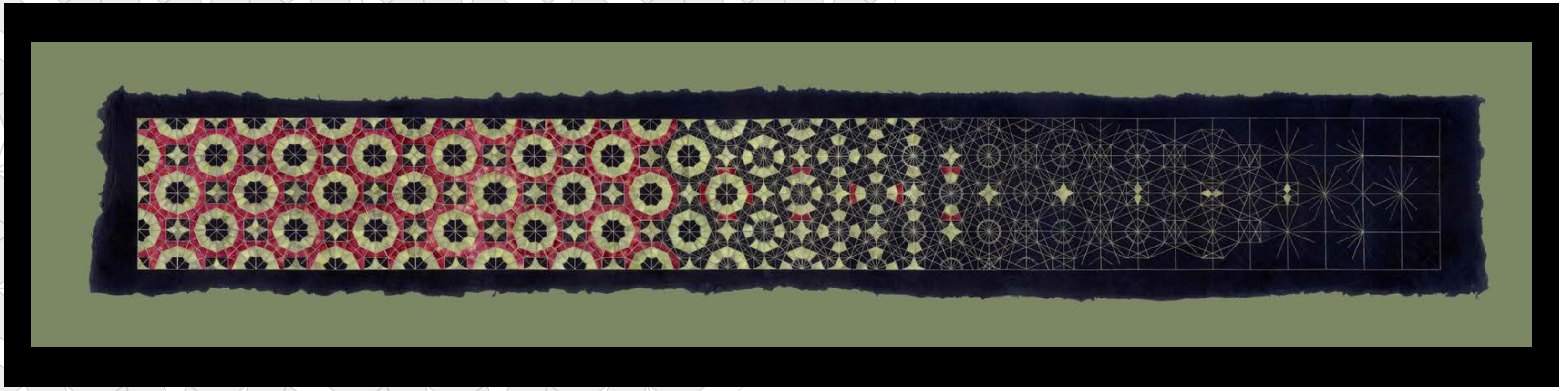
Hasret frequently runs workshops at community events, public engagements, galleries, education centres and school project settings as well as her public and private commissioned works. Her work is sold internationally and exhibited across the UK, and featured in the Quarter Life publication as a working artist in Birmingham's Jewellery Quarter. In 2017 she was a nominee and finalist at the British Muslim Awards for her contributions in the Arts and Culture Awareness category.

Currently her work is in the process of taking an interesting direction which fuses this traditional art form with her western influences and will be revealed as new collection sometime in 2022.



me@hasretbrown.com  
HasretBrown.com





**Hasret Brown**



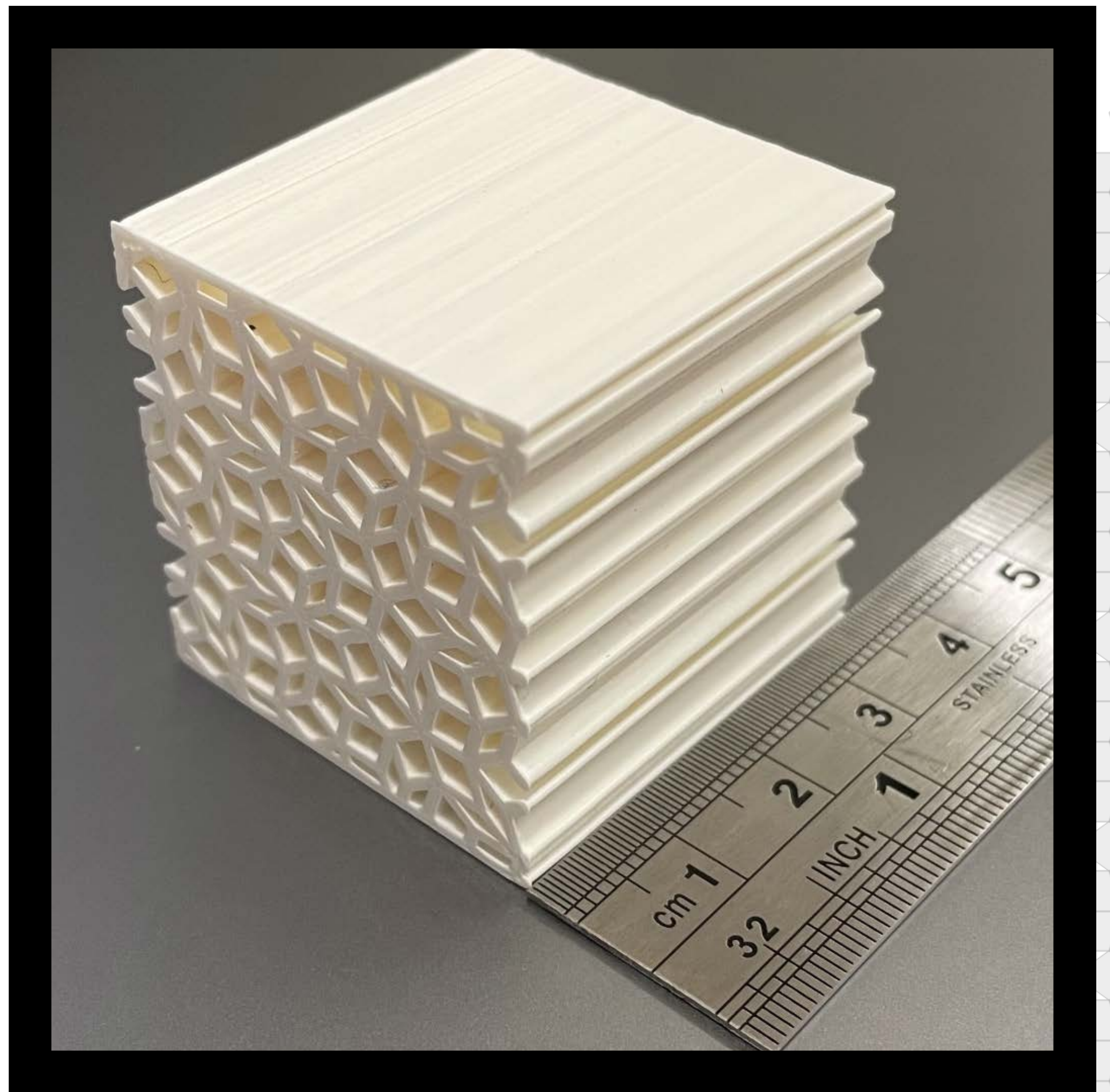
**Hasret Brown**



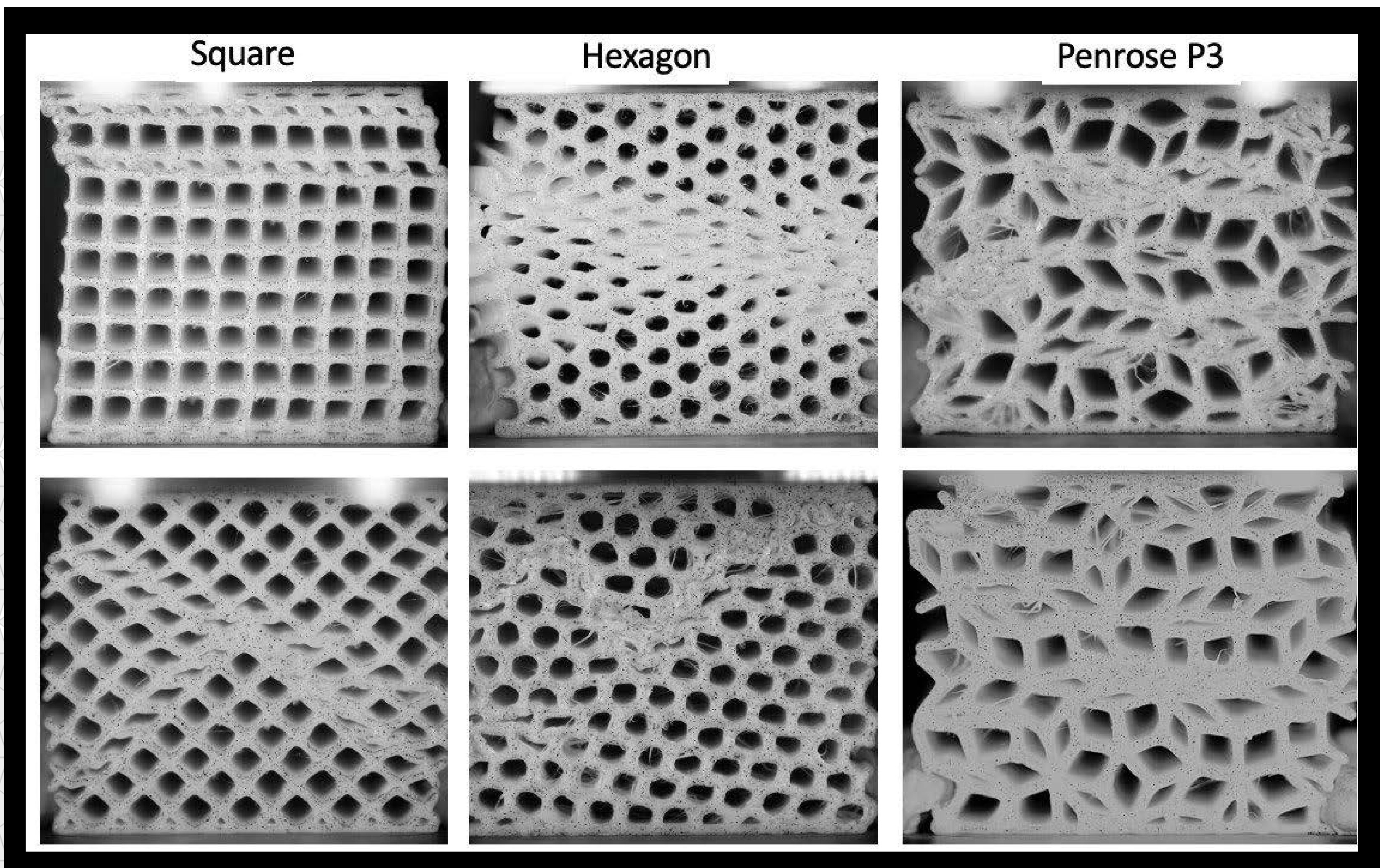
# **RICHARD MOAT, IESTYN JOWERS, CHIKWESIRI (CK) IMEDIEGWU, DANIEL CLARKE, & PATRICK CARTER**

We are a multidisciplinary research team exploring mechanical behaviours of designed components with internal structures that exhibit aperiodic order. Such components have beams in place to carry necessary forces but empty space elsewhere. The geometries of aperiodic patterns reduce anisotropies which give rise to unwanted behaviours, and which impact the performance of resulting structures. They achieve this whilst retaining the benefits of being deterministic, resulting in materials that are isotropic, predictable and scalable. Such materials have the potential for use in high-risk applications such as automotive, aerospace and medical engineering, where unforeseen anisotropies or defects may have dire consequences.





**Group**



**Group**



# JOHN HUNTON

John is currently Professor in Pure Mathematics at Durham University, having previously held posts in Cambridge, St Andrews, Edinburgh, Manchester and Leicester. He was part of the team, including Uwe, who put on the exhibition “How do Shapes Fill Space” at the Royal Society Summer Science Exhibition 2009. He has long been interested in the dissemination of mathematics both within the academic world and to the wider community. For many years he ran the London Mathematical Society’s portfolio of 12 journals and 2 book series.



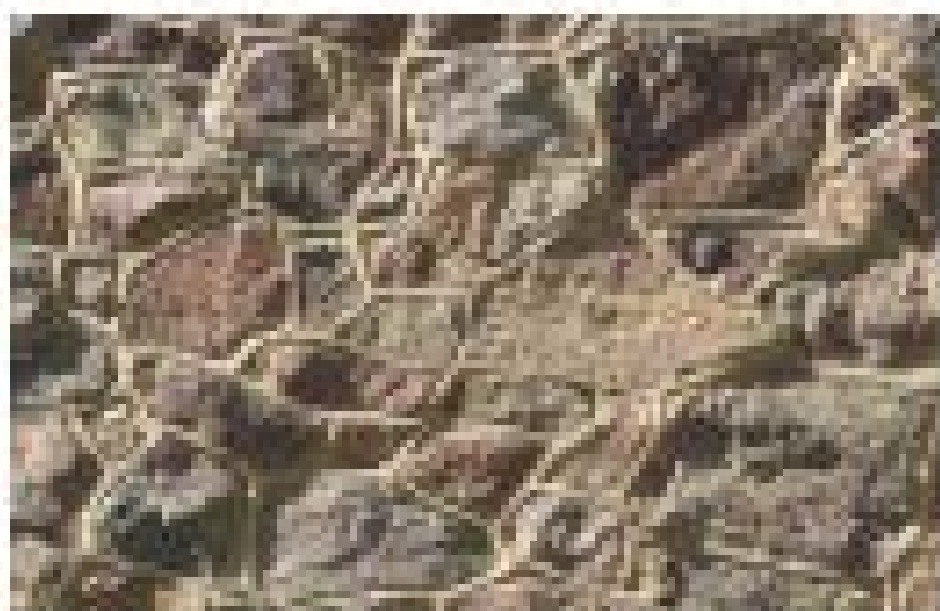
[john.hunton@durham.ac.uk](mailto:john.hunton@durham.ac.uk)  
[www.dur.ac.uk/john.hunton](http://www.dur.ac.uk/john.hunton)



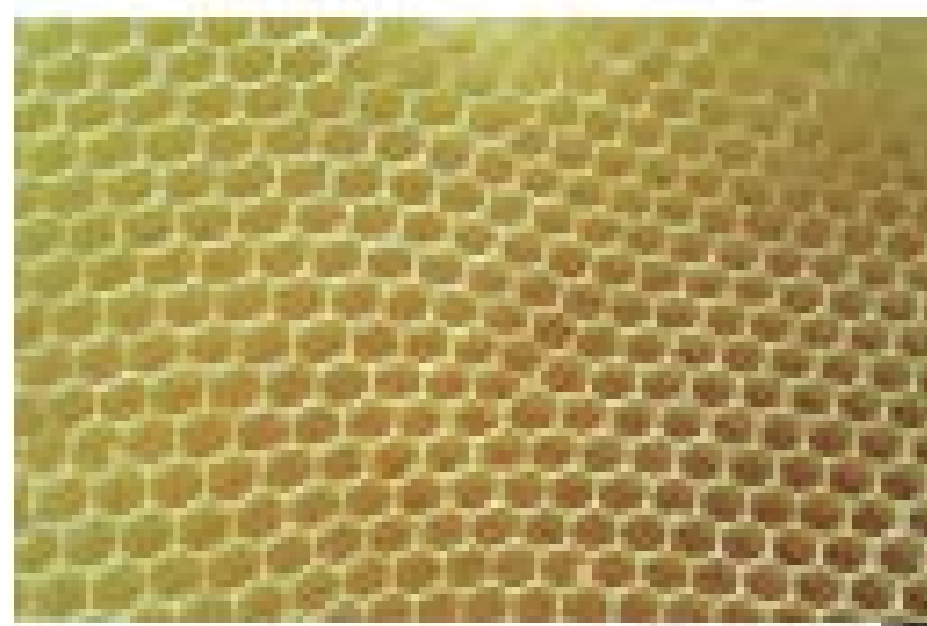


# Where do we start?

From ancient times people have been putting shapes together, both for practical purposes and for decoration. They also appear in many ways in the natural world.



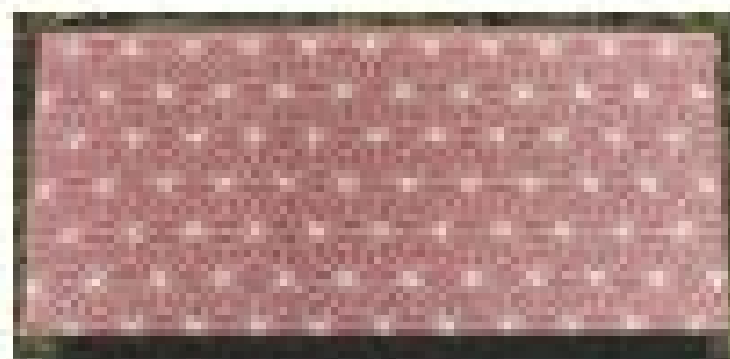
**Stone Walling**  
Great Malvern, England



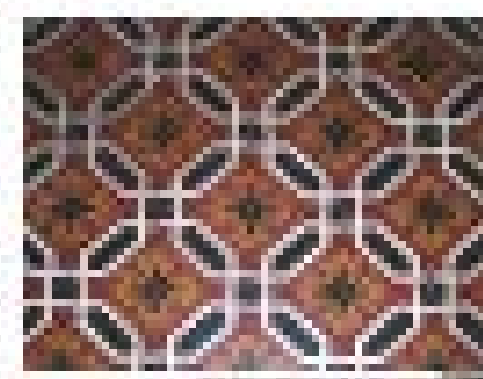
**Honeycomb**  
Black Beehive, France



**Fairisle Jumpers**  
Fairisle, Scotland



**Fabric Design**  
Kirikozeno, Japan



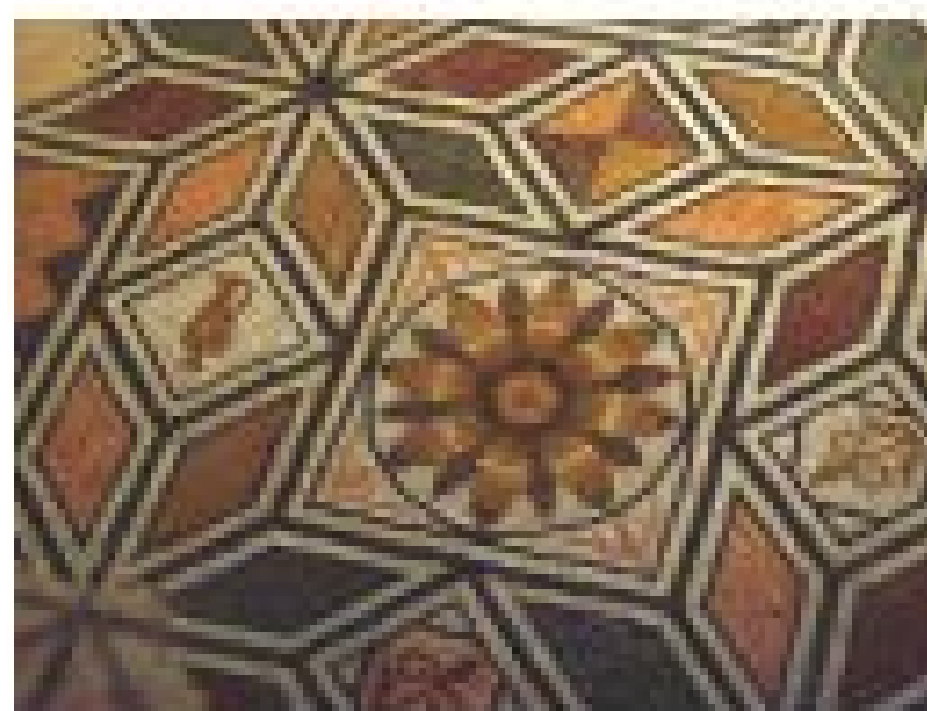
**Church Tiling**  
Caisleá, Maseo Grossa, Brazil

Tilings and shapes have been part of mathematics since its birth, appearing in Euclid's *Elements* (c.300 BC) the earliest known maths book. Mathematics aims to understand the underlying rules that govern and limit things. Two key tools for this are:

**Classification:** can we list all possible objects?

**Generalisation:** can we take my understanding of a specific case and apply it to a wider setting?

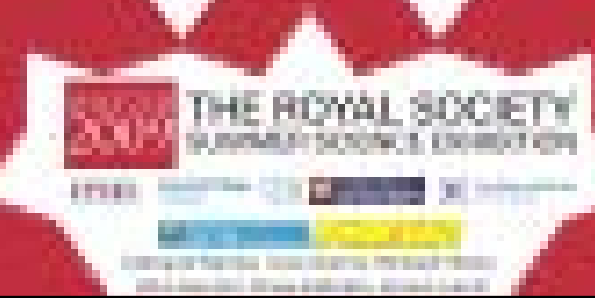
Can we classify all tilings? Even on the plane this is too difficult, let's simplify it by restricting the set of shapes. Can you tell which of the regular shapes below can fill the plane? We can generalise this to other tilings of the plane, but also to higher dimensions. We can even generalise 'space' and classify possible 'spaces' which shapes could fill.



**Roman Mosaic**  
Domus dell'Ortaglia, Brescia, Italy



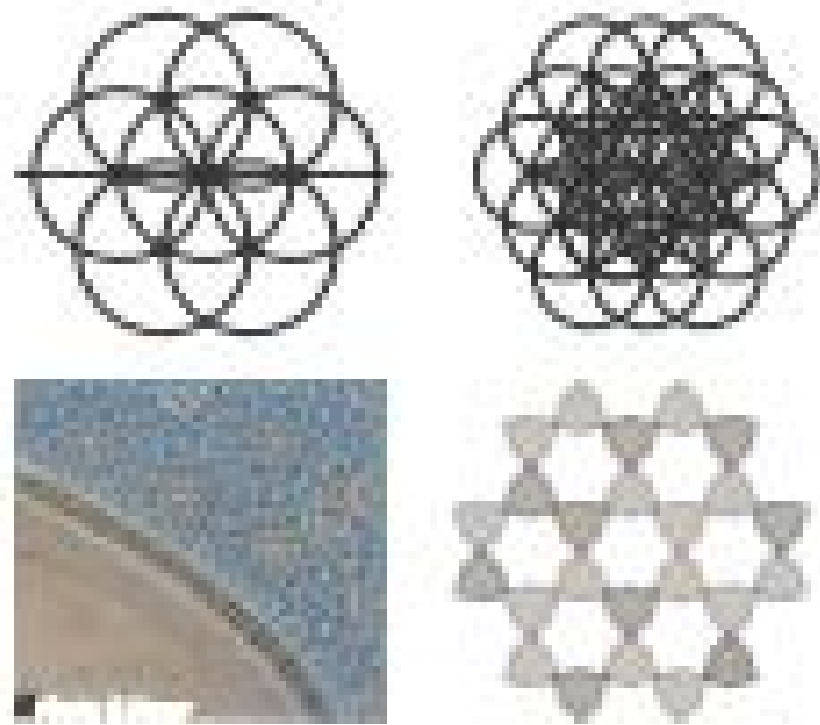
In this exhibit we look at ways shapes fit or don't fit together. Starting with simple shapes, we trace some of the ways they have been used in art and mathematics throughout history, and look at the problems and ideas that have arisen. This leads to current research problems in mathematics, and their consequences for science.



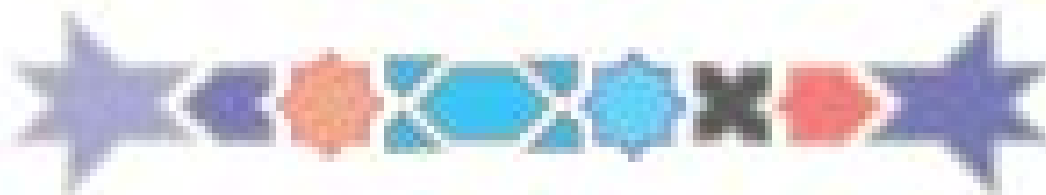
John Hunton



# What are the secrets of the Islamic master craftsmen?



The use of geometry creates a natural harmony within Islamic art. This relates to the Islamic belief that all creation is harmoniously interrelated. The pattern above (15th century) from Yazd in Iran is derived from regular divisions of the circle. A regular grid of triangles is established, on top of which the design is elaborated.



A family of *zilli* shapes, derived from the octagram (*shahmeh* – 8-pointed star). These are typical of the ceramic, cut-tile pieces used in mosaics throughout Anatolia and Morocco from the early medieval period onwards.



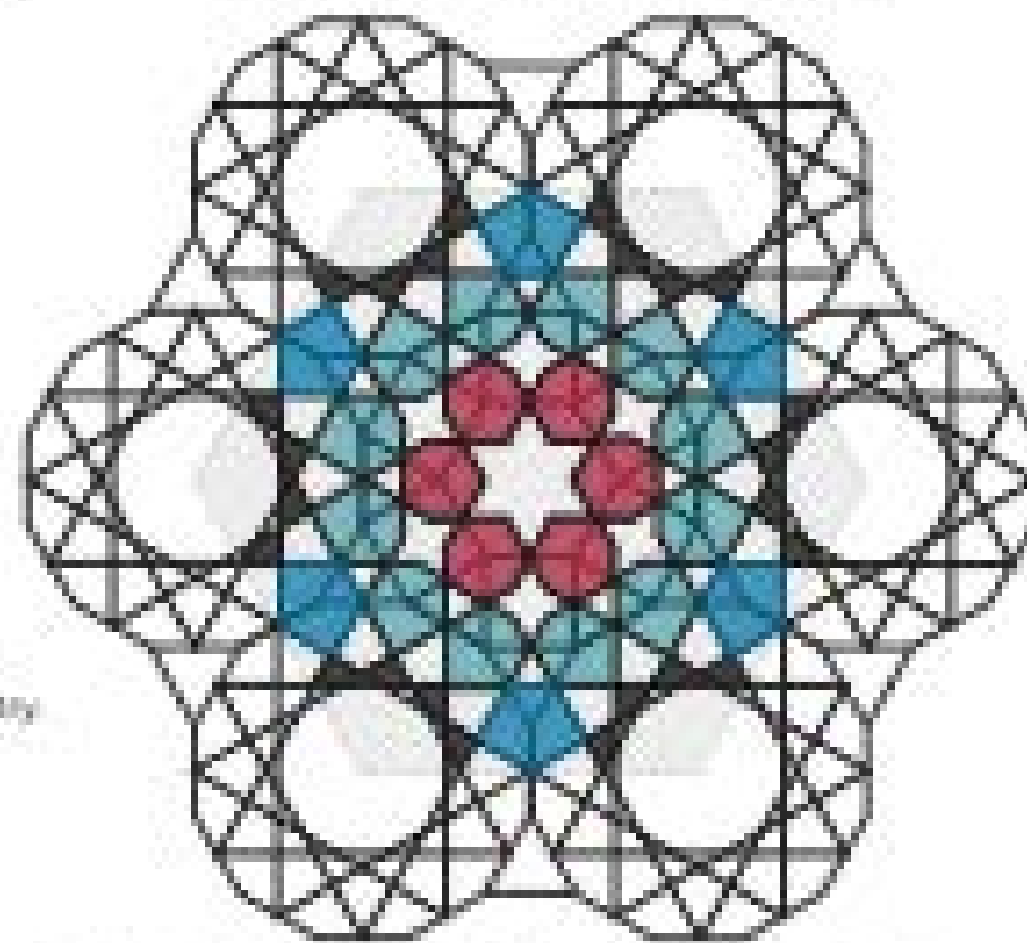
In the Alhambra (1485 C), Spain (above), geometric pattern is perfectly integrated with biographic design (*arabesque*) and calligraphy. These decorative elements are deployed using a range of symmetries that have now been classified as belonging to distinct mathematical groups.



With a compass and ruler an Iranian master craftsman demonstrates a method for setting out a pattern which employs 10-fold radial symmetry. Note the construction of the decagram (10-pointed star).



Four examples of how to derive classic motifs from the decagram. These form the basis of 12th century Iranian *gash* patterns, which have been linked to Penrose's famous tilings.

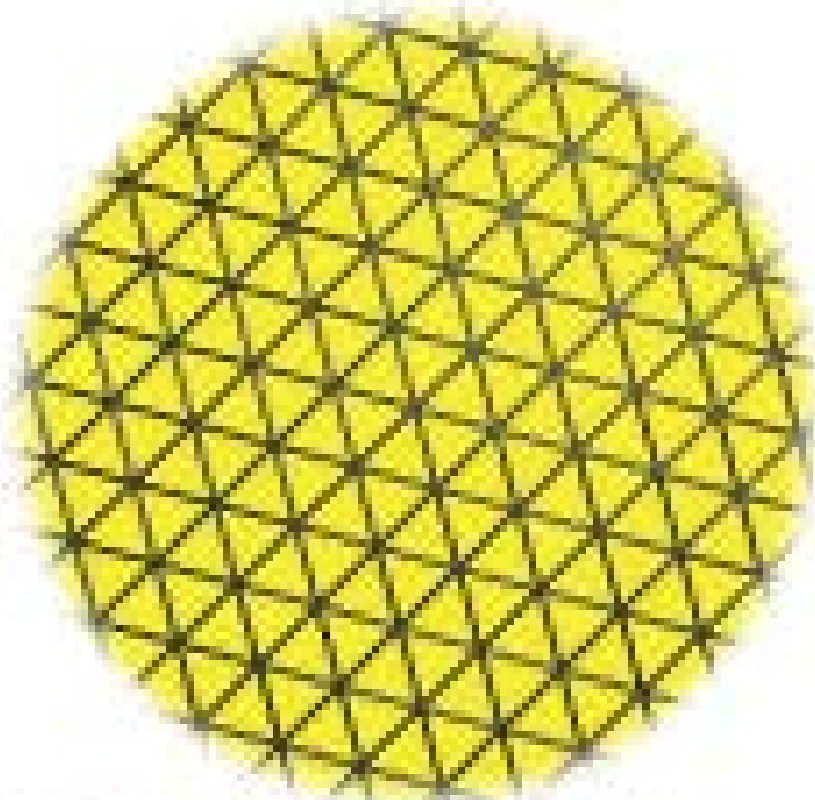


Islamic designers used the full range of Archimedean tilings to explore pattern development. The pattern above is based on elements of medieval designs, typically found in Cairo. Can you understand the underlying structure?



# How can triangles meet?

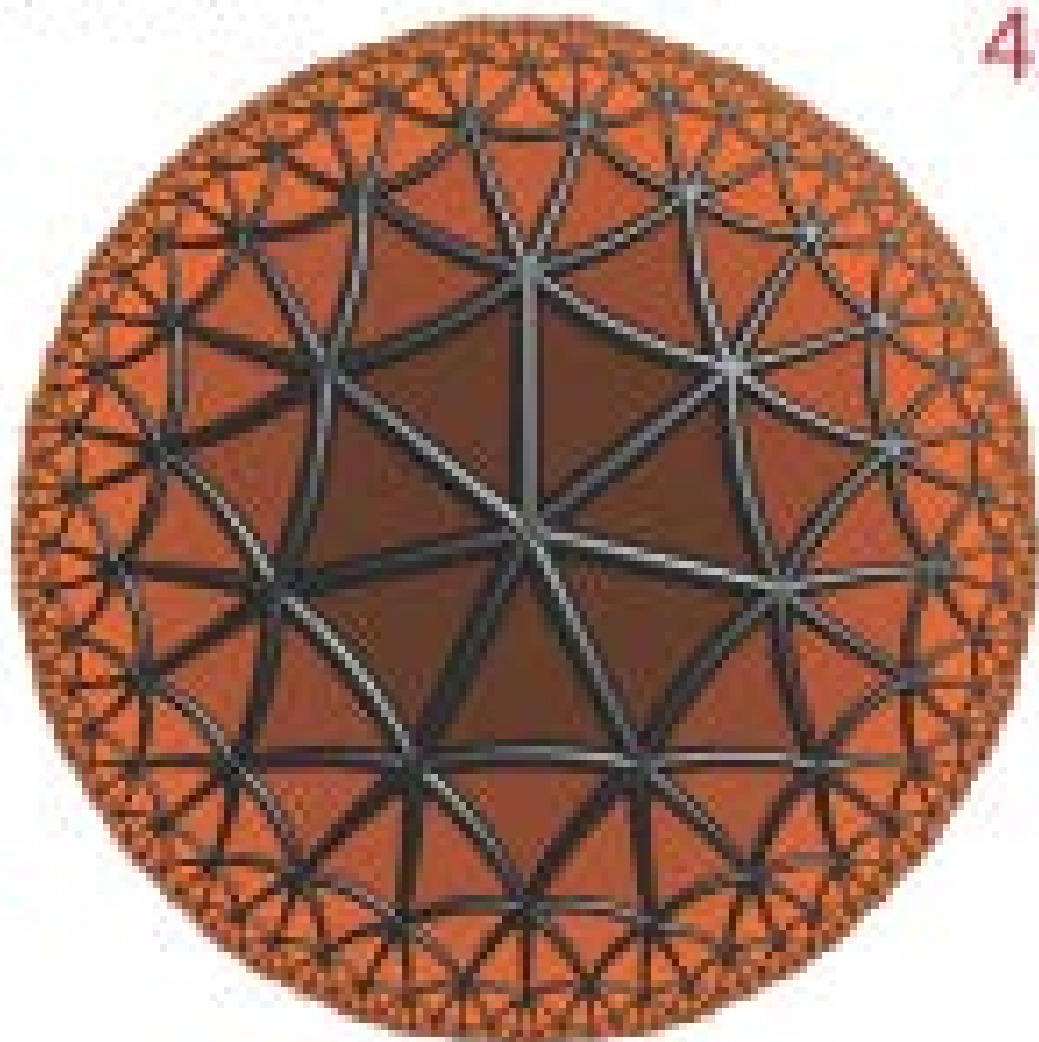
Six equilateral triangles lying flat round a point meet up. This can be extended to fill the plane:



## 6: Euclidean Tiling

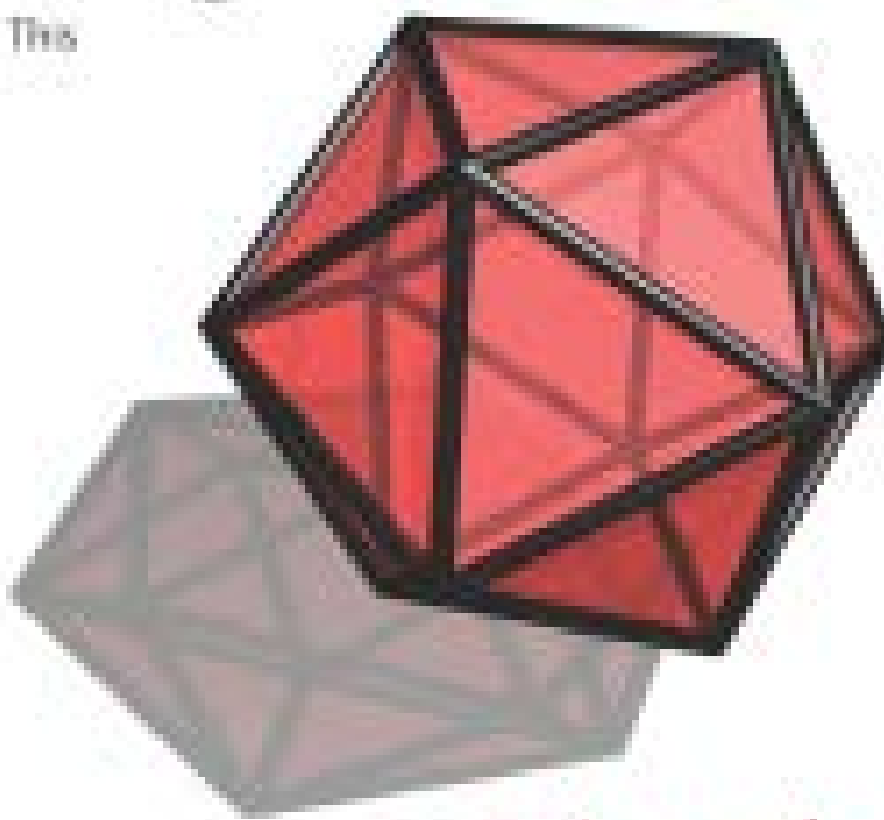
Infinite triangles  
6 round a vertex

More than six equilateral triangles can meet up, but we have to fold in a different way. Can this be extended to a surface? Yes if you use mysterious hyperbolic space:



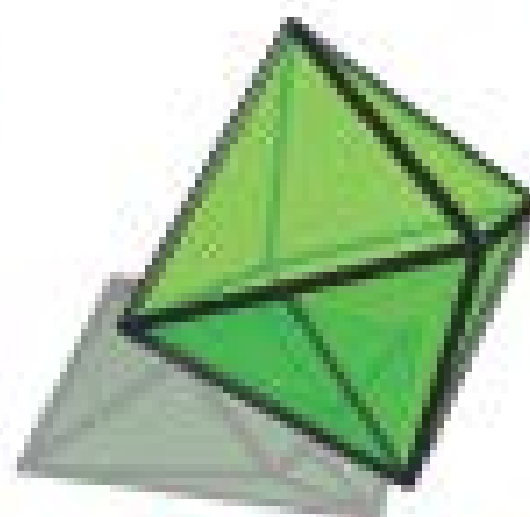
## 7: Hyperbolic Tiling

Infinite triangles  
7 round a vertex



## 5: Icosahedron

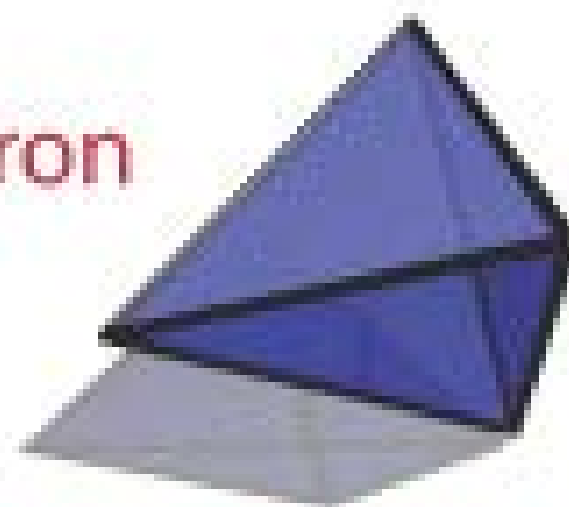
20 triangles  
3 round a vertex



## 4: Octahedron

8 triangles  
4 round a vertex

Fewer than six triangles do not meet up if they lie flat. They can meet if we fold into a third dimension. Extending this gives three 3-dimensional shapes.



## 3: Tetrahedron

4 triangles  
3 round a vertex

Can you see what happens when you put 3, 4 or 5 squares round a point? What about other polygons?

### Johannes Kepler 1571-1630

Johannes Kepler was a German mathematician and astronomer. He discovered laws of planetary motion that helped Newton to discover gravity. He was fascinated by polyhedra, even proposing them as a model for the body of man. He made lots of things including the Alchymical Triangle and a surprising image which was the Flower of Life.



2009 THE ROYAL SOCIETY SUMMER SCIENCE COLLECTION

1998 2000 2002 2004 2006 2008

2009 2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023 2024 2025



# How can 3d shapes meet?

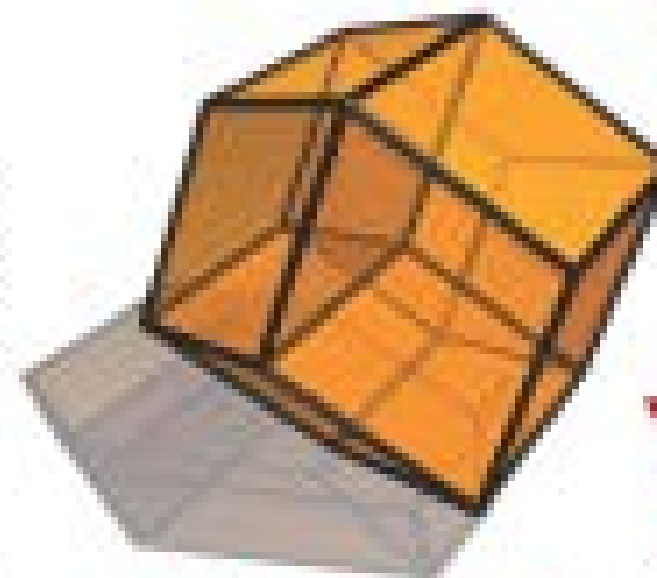
Just as we can look at 2d shapes round a point, we can consider 3d shapes round an edge. Four cubes round an edge meet up. This can be extended to fill 3d space:



## Cubic Lattice

Infinite cubes  
4 round each edge

Three cubes round an edge do not meet up, but we can fold again, into a fourth dimension, to give the Tesseract. We can't see this but we can see 3d shadows.



## Tesseract

8 cubes  
1 round each edge  
(4d version of square and cube)



## 24-cell

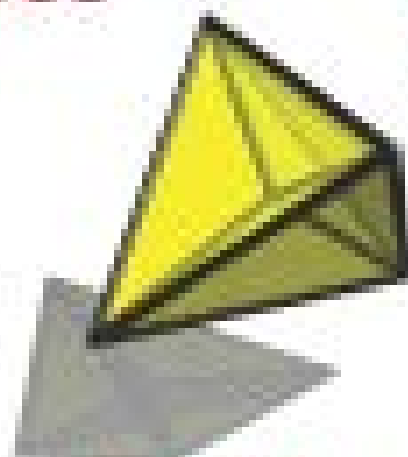
24 octahedra  
1 round each edge

Using these ideas and mathematical imagination we can find all possible regular shapes. There are six in 4d, but only three in each higher dimension, can you work out what they are?



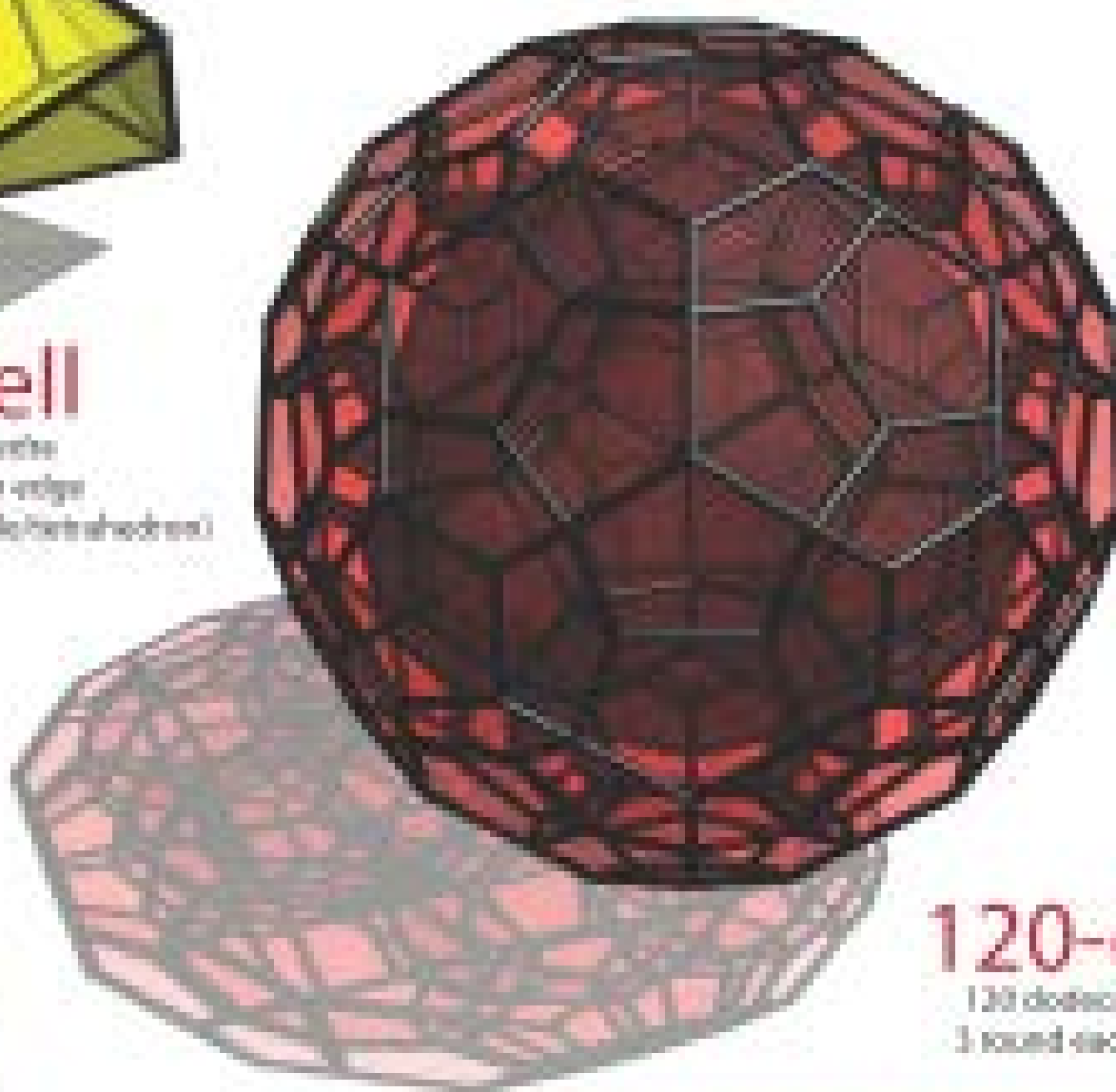
## 16-cell

16 tetrahedra  
4 round each edge  
(4d version of octahedron)



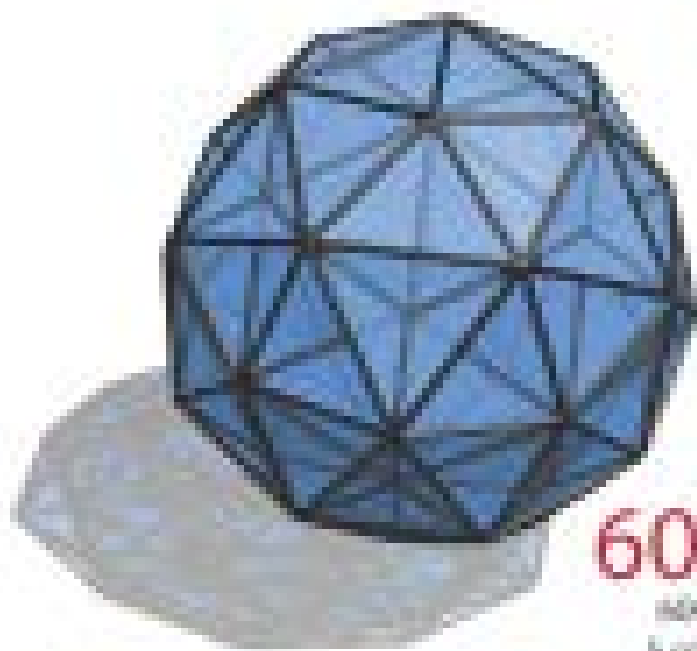
## 5-cell

5 tetrahedra  
3 round an edge  
(4d version of triangle/tetrahedron)



## 120-cell

120 dodecahedra  
1 round each edge



## 600-cell

600 tetrahedra  
5 round an edge

### Alicia Boole Stott 1860-1940

Alicia Boole Stott was a British mathematician and daughter of the mathematician George Boole. She had a phenomenal intuition for higher dimensional spaces, finding the six regular polytopes in 4d and proving there were no others. She was able to make models of their three-dimensional cross-sections.

THE ROYAL SOCIETY  
SUMMER SCIENCE EXHIBITION

2018

2019

2020

2021

2022

2023

2024

John Hunton



**What do you see?**

**Roger Penrose 1931-**  
 Sir Roger Penrose FRS is an English mathematical physicist. He is renowned for his work in mathematical physics, in particular for contributions to general relativity and cosmology. He also made an important contribution to string theory, by discovering a set of ten shapes that can tile the plane only in a non-periodic way (see this poster).

**THE ROYAL SOCIETY**  
 MATHEMATICAL PHYSICS SECTION

**John Hunton**



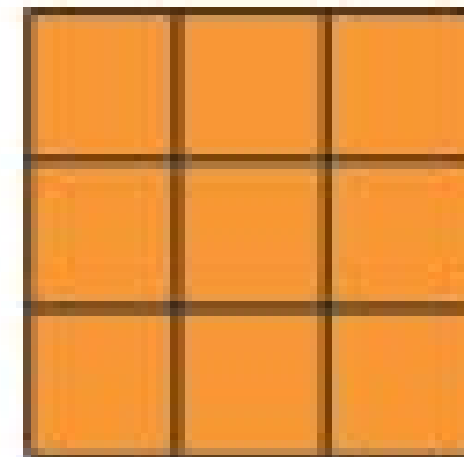
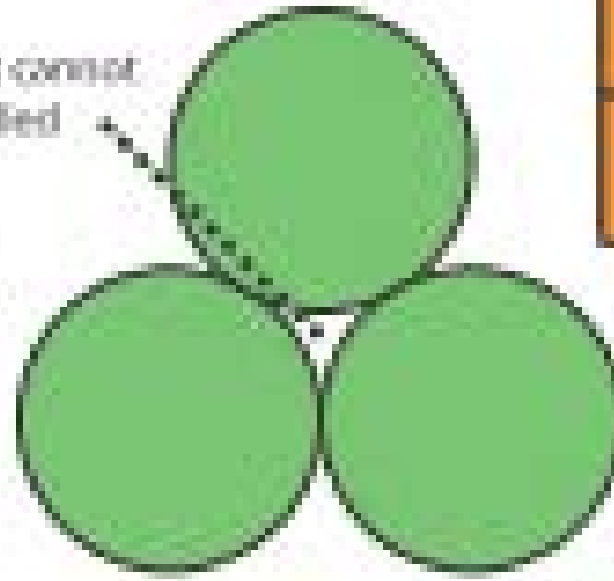
# What shapes fill the plane?

If I give you some shapes can you tell me whether they can tile the whole plane?

## Easy Examples

For some collections of shapes the answer is easy. For example circles cannot fill the plane without leaving gaps, whereas squares fit together exactly.

Hole that cannot be filled

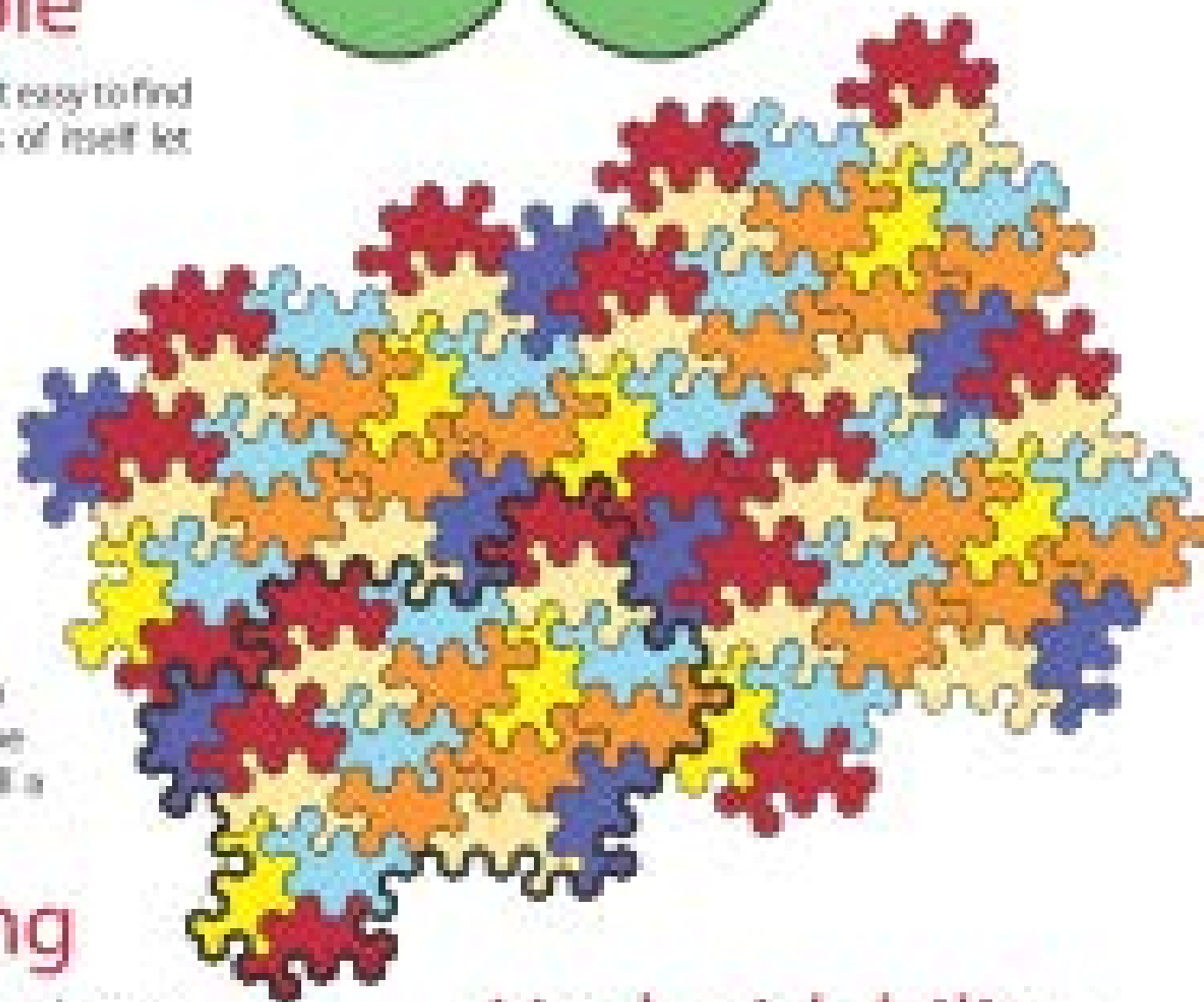


## Harder Example

This shape, the Myers tile, is harder, it is not easy to find ways that it can fit together with copies of itself let alone fill the plane.



In fact it can tile the plane as shown on the right. Could you have found this tiling just from the tile?



## Periodic Tiling

Both the square tiling and the Myers Tiling are called periodic. For the square, a single tile gives a region that repeats. The Myers Tiling is more complicated, but still a region of tiles is simply repeated.

## Aperiodic Tiling

Can all collections of shapes that can fill the plane produce a periodic tiling? The answer is NO! There are strange sets of Aperiodic shapes that can fit together to fill the plane, but never periodically. An example are the two Penrose tiles:



As you can see from the last poster these can fit together to cover a large area.

## Monotile Problem

All known aperiodic sets of tiles have at least two shapes. Is there a single aperiodic tile? No one knows!

## Undecidability

Aperiodic tiles are an example of how things can start to get complicated. In fact things can get a lot wilder. The question of whether a set of shapes can fill the plane can be answered in many cases, however no computer algorithm can say YES or NO for every collection of shapes. In this way it is an example of an undecidable problem, beyond the power of computers.

### Alan Turing 1912-1954

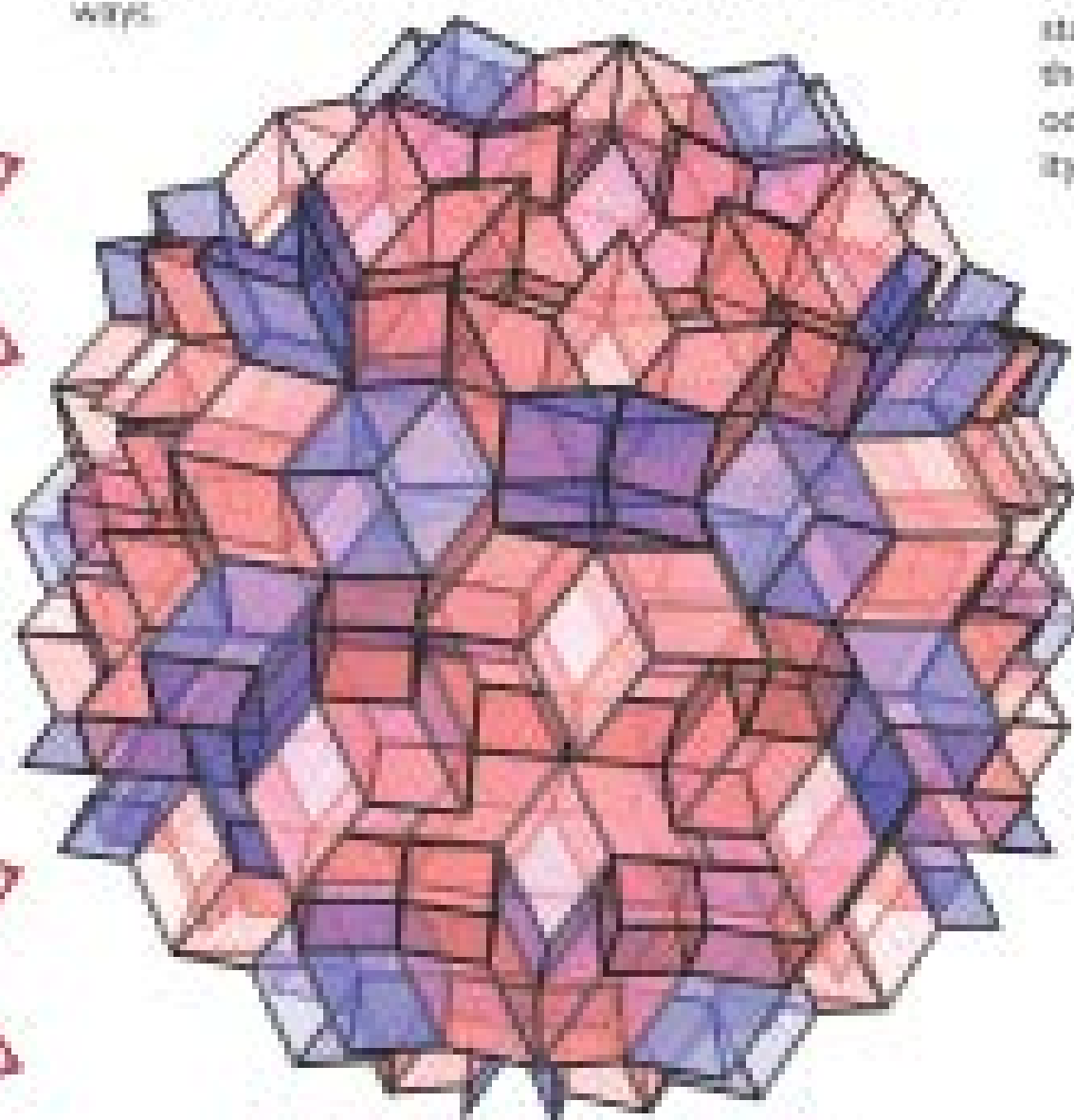
Alan Turing FRS was a British mathematician and pioneering computer scientist. During the second world war he was instrumental in breaking German codes at Bletchley Park. With the history of Turing machines he established the foundations and theory of computer science. He also devised the Turing test to decide whether a machine displays intelligence.





# New Maths, New Science!

Aperiodic tilings were first discovered in the 1960's. It is now known that there are an infinite number of different examples. The study of such tilings has become a new branch of mathematics and is developing in several ways.



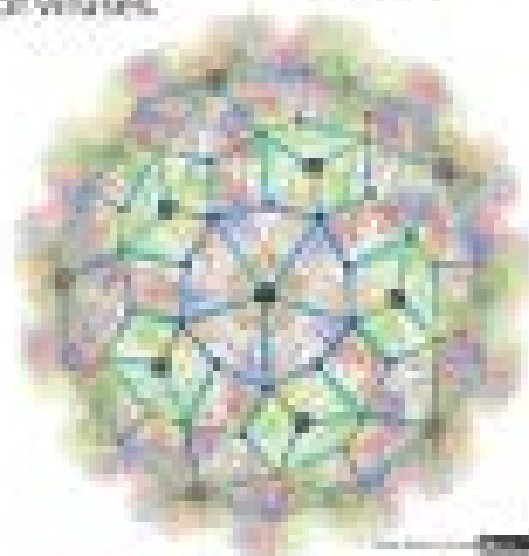
## Icosahedral Tiling

A 3d version of the Penrose Tiling

## New Science

It is often the case that scientific discoveries have to wait until mathematics has provided the ideas and language needed to recognise them. Several years after their discovery, aperiodic tilings were found in nature, marking the positions of atoms in quasicrystals. More recently they are also being used to understand and predict the behaviour of viruses.

The atomic structure of solids, such as quasicrystals is detected by diffraction experiments. Research carried out by members of the exhibiting team is addressing questions such as: What diffraction patterns do aperiodically ordered materials have? Can one actually distinguish order from disorder?



## Virus Structure

Icosahedral structure used to analyse the proteins in a virus

## New Maths

Can you list all possible aperiodic tilings? Can you understand all aperiodic tilings with certain properties, such as those which are the shadows of high dimensional periodic tilings, or those which have fractal-like self similarity?

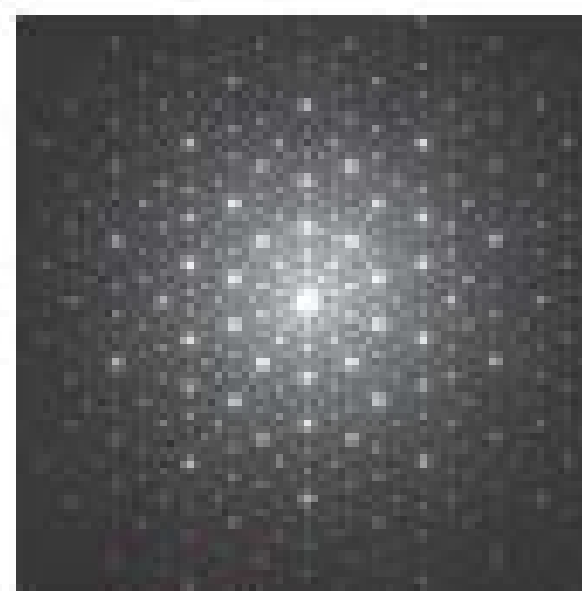
How different can aperiodic tilings be? How dependent are the patterns formed on the exact shapes of tiles used? How can we characterise an infinite, never repeating pattern?

The researchers who put on this exhibit, their collaborators from all around the world and other mathematicians, have developed ways of answering such questions, but many puzzles still remain unsolved. Mathematics always needs new minds and new ideas. Mathematics needs you!



## Quasicrystal

Electron microscope image of the surface of a collection of quasicrystals



## Diffraction

Diffraction Pattern for a quasicrystal. Note the five fold rotational symmetry that cannot occur for a periodic crystal.



# KURT HOFSTETTER

Born 1959 in Linz, Austria, Kurt Hofstetter lives and works in Vienna, Austria.

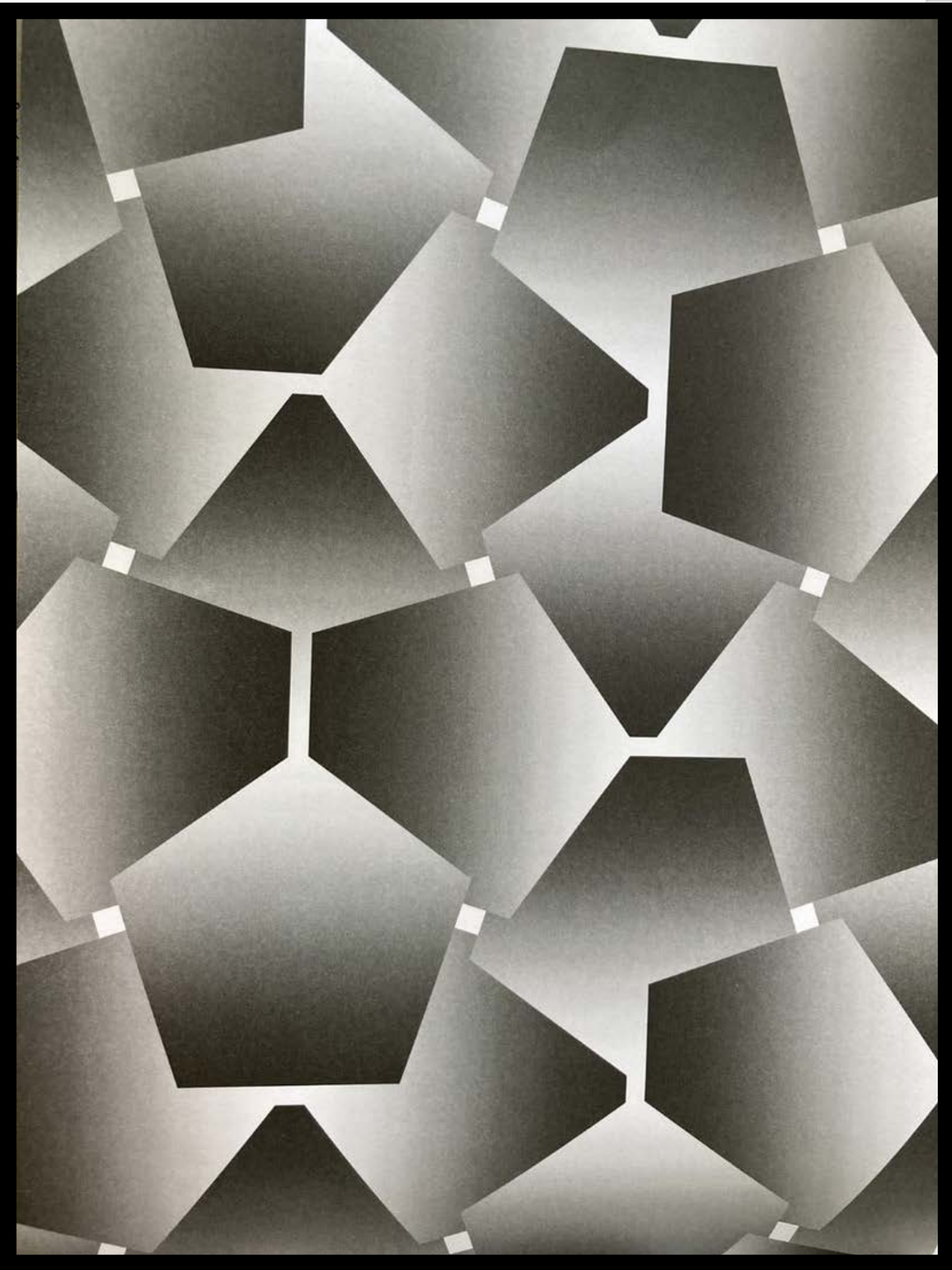
Artistic domains:

Concepts, Public Sculptures, Media Art Installations, Ambient Media Art, Music Compositions, Art Videos, Mathematical Reflections, Irrational Patterns, Ambient Tactile Art, Artistic Research since 1998 artist community with Barbara Doser "Parallel Media" in the areas of video and video installation

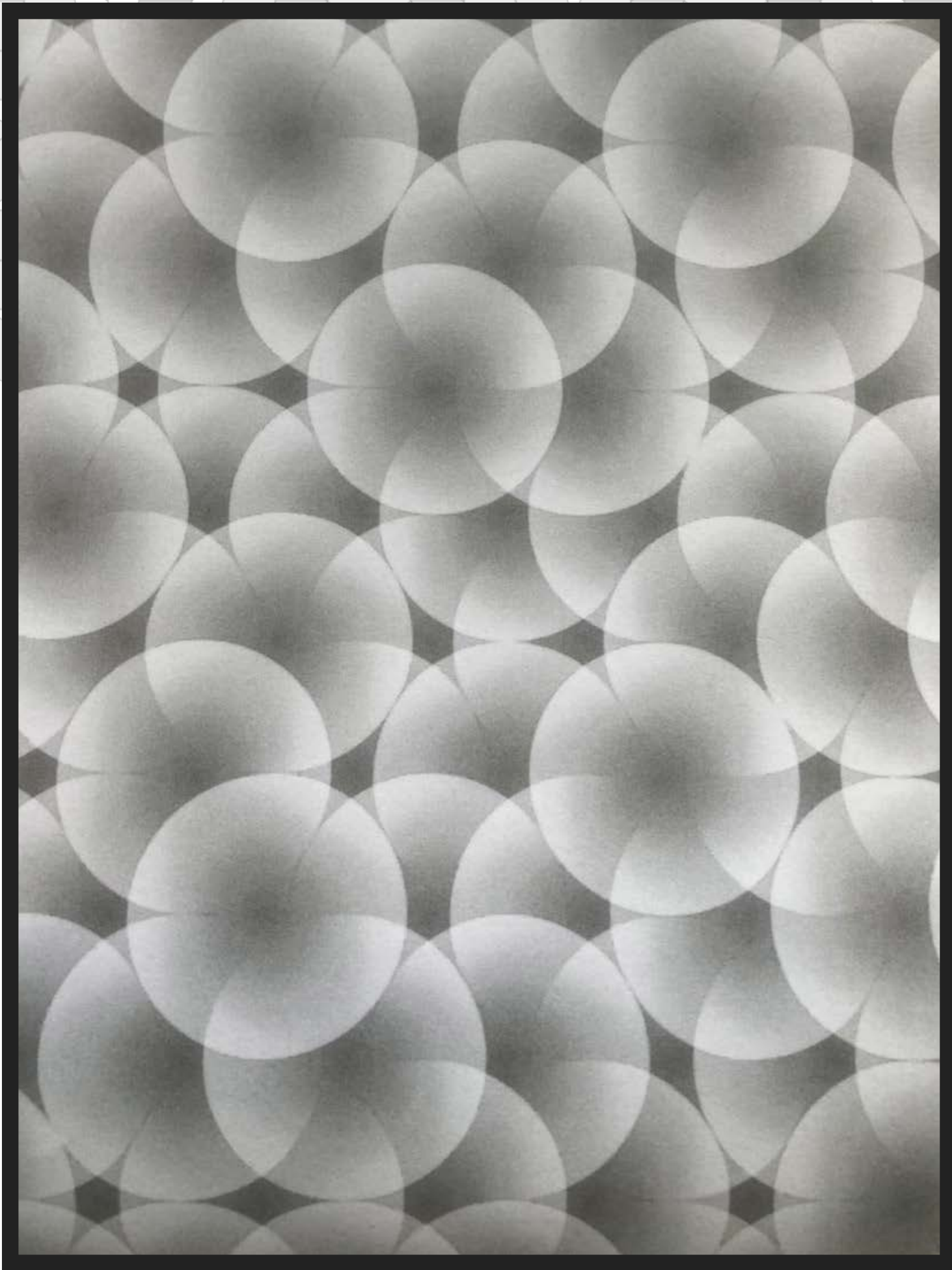
2008, Discovery of the Inductive Rotation, a new recursive method to create aperiodic and asymmetric tilings and structures. 2015, State Prize of the Republic of Austria for the project On the Event Horizon of Order - New Irrational Patterns. 2020, State Prize of the Republic of Austria for the extensive, internationally recognized oeuvre



[hofstetterkurt.net](http://hofstetterkurt.net)

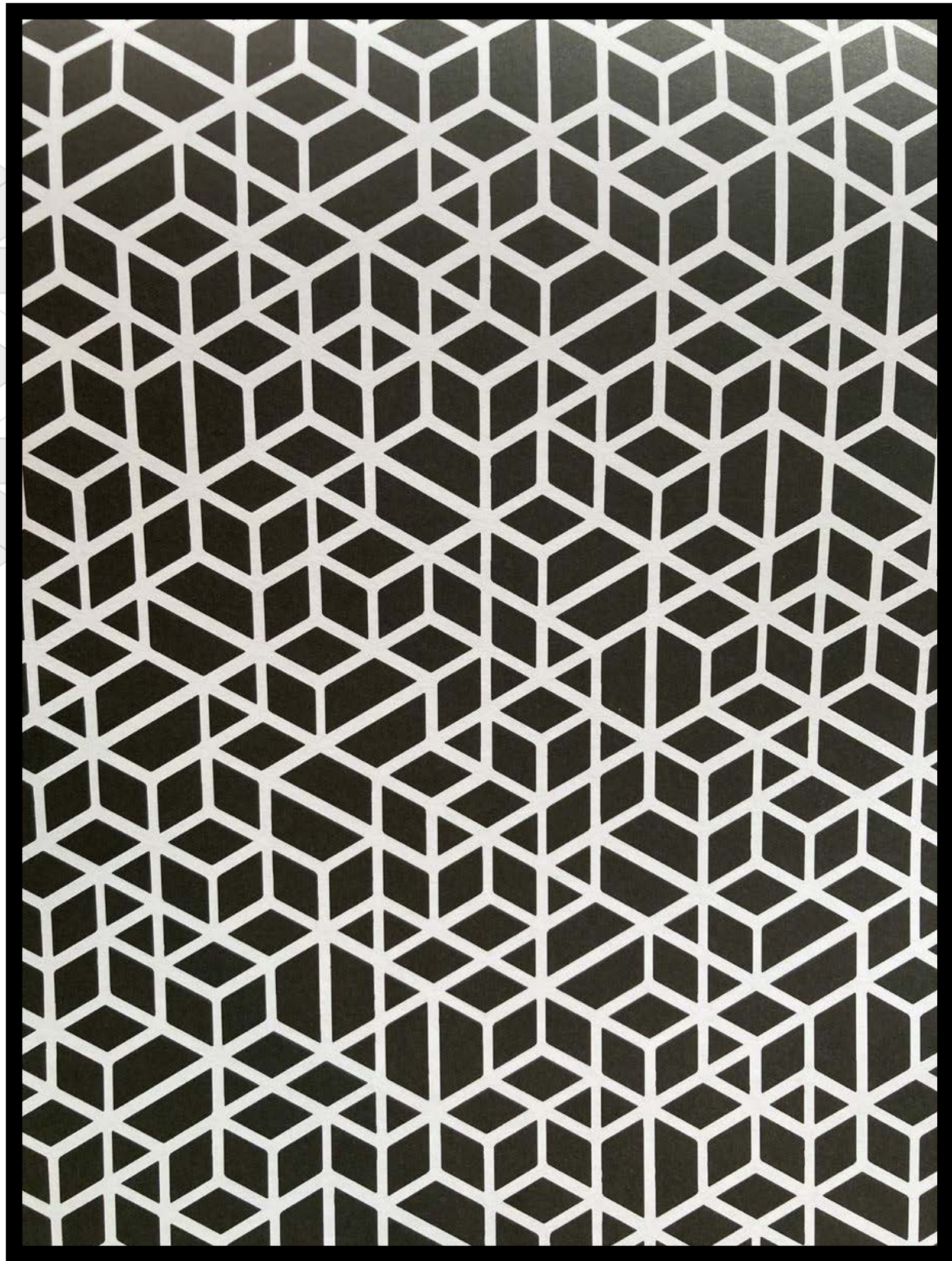
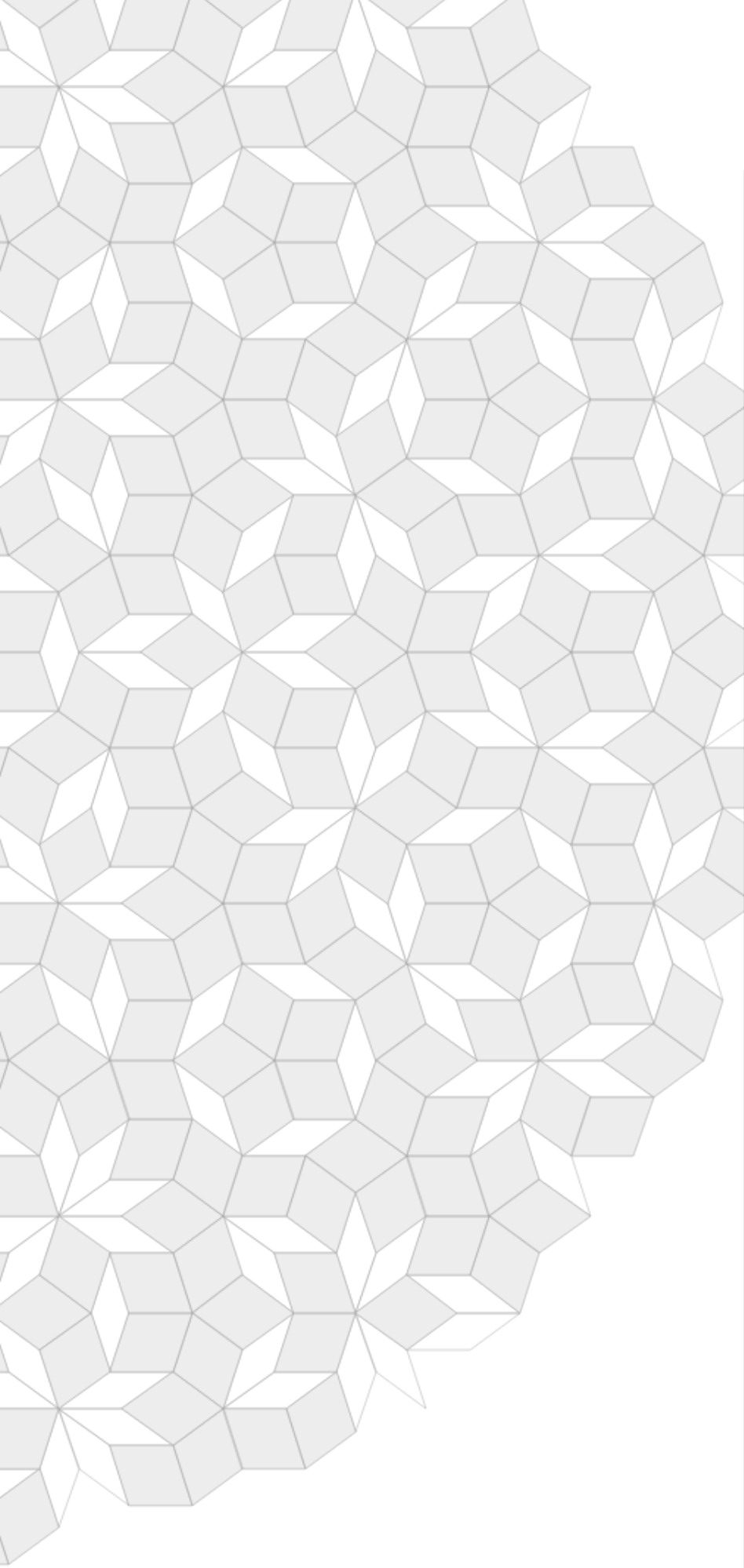


**Kurt Hofstetter**

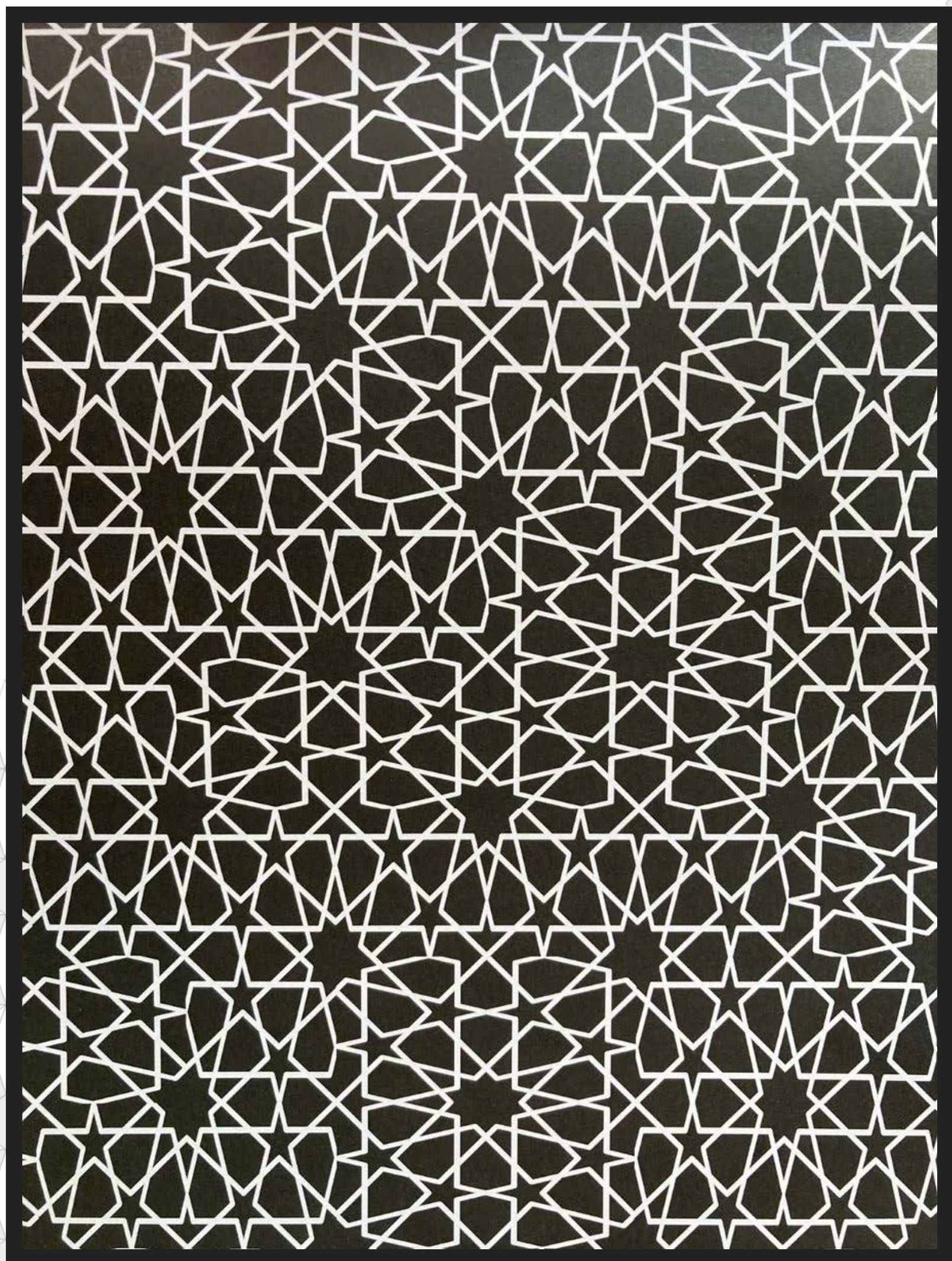


**Kurt Hofstetter**

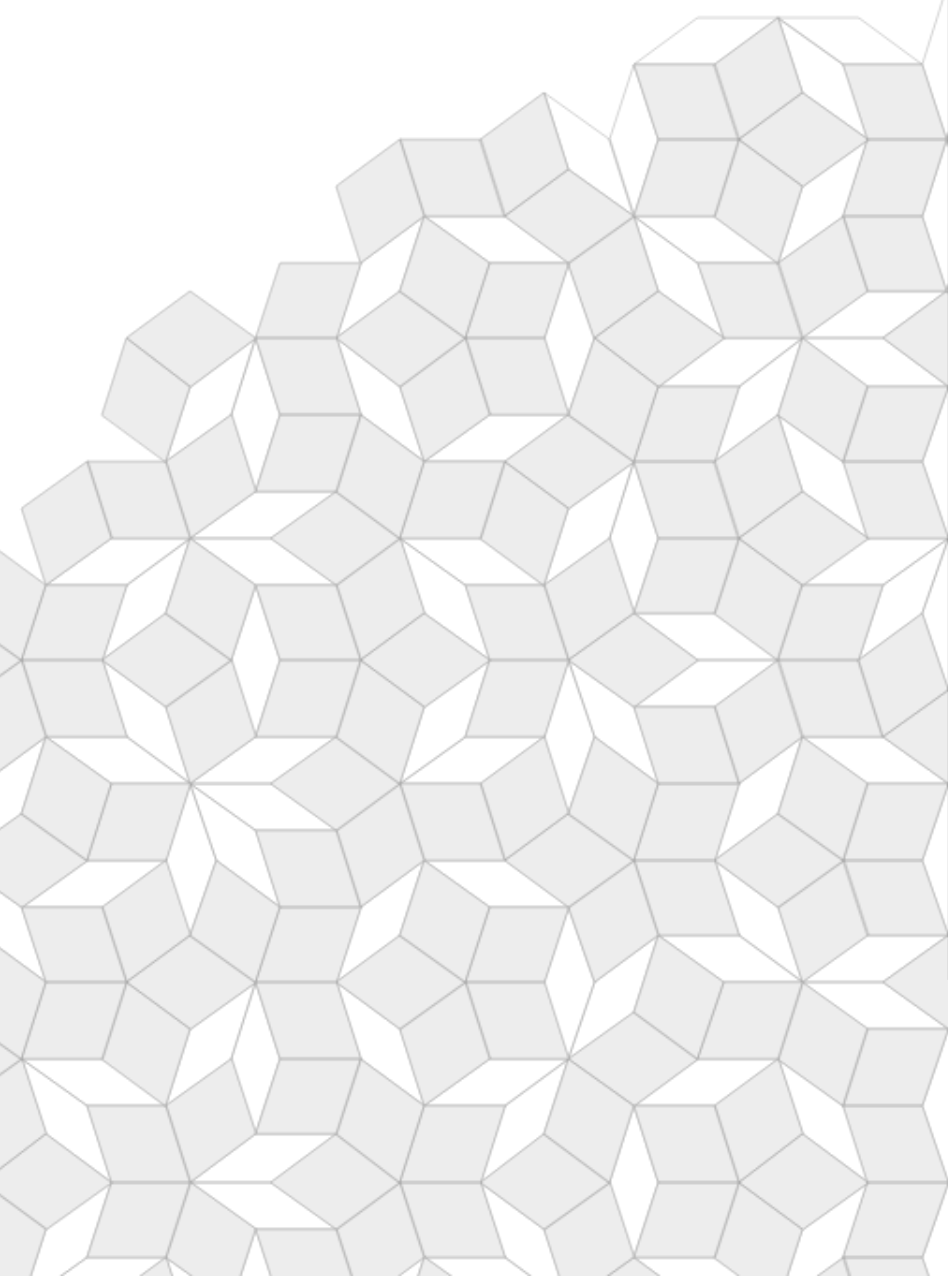
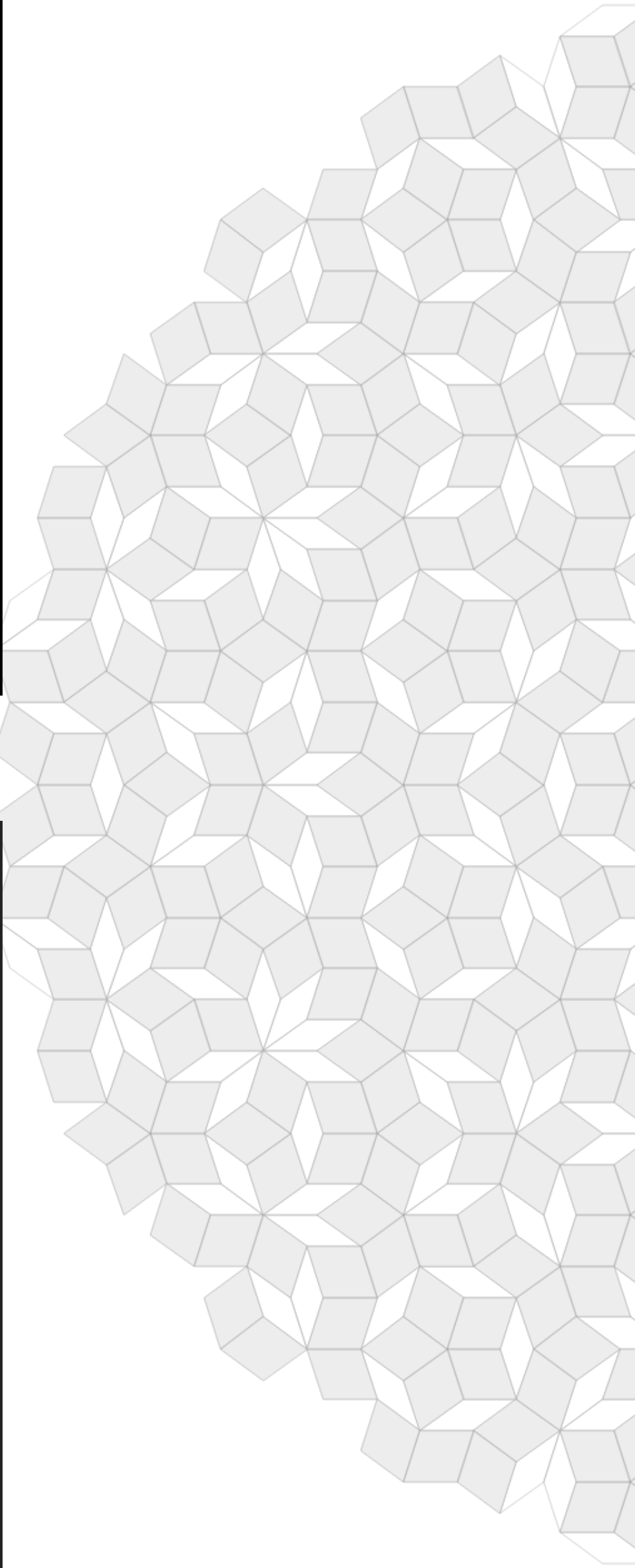




**Kurt Hofstetter**

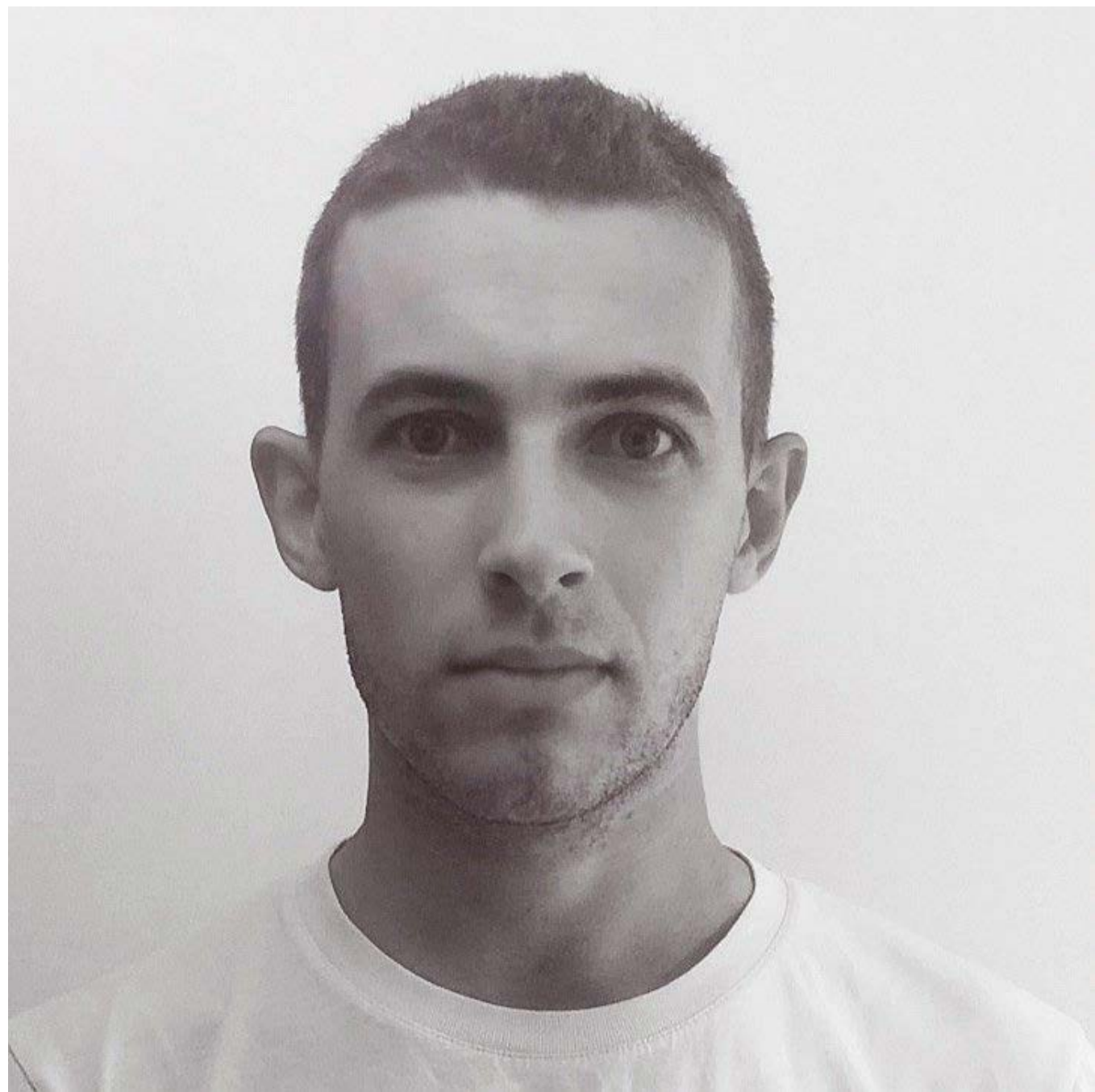


**Kurt Hofstetter**



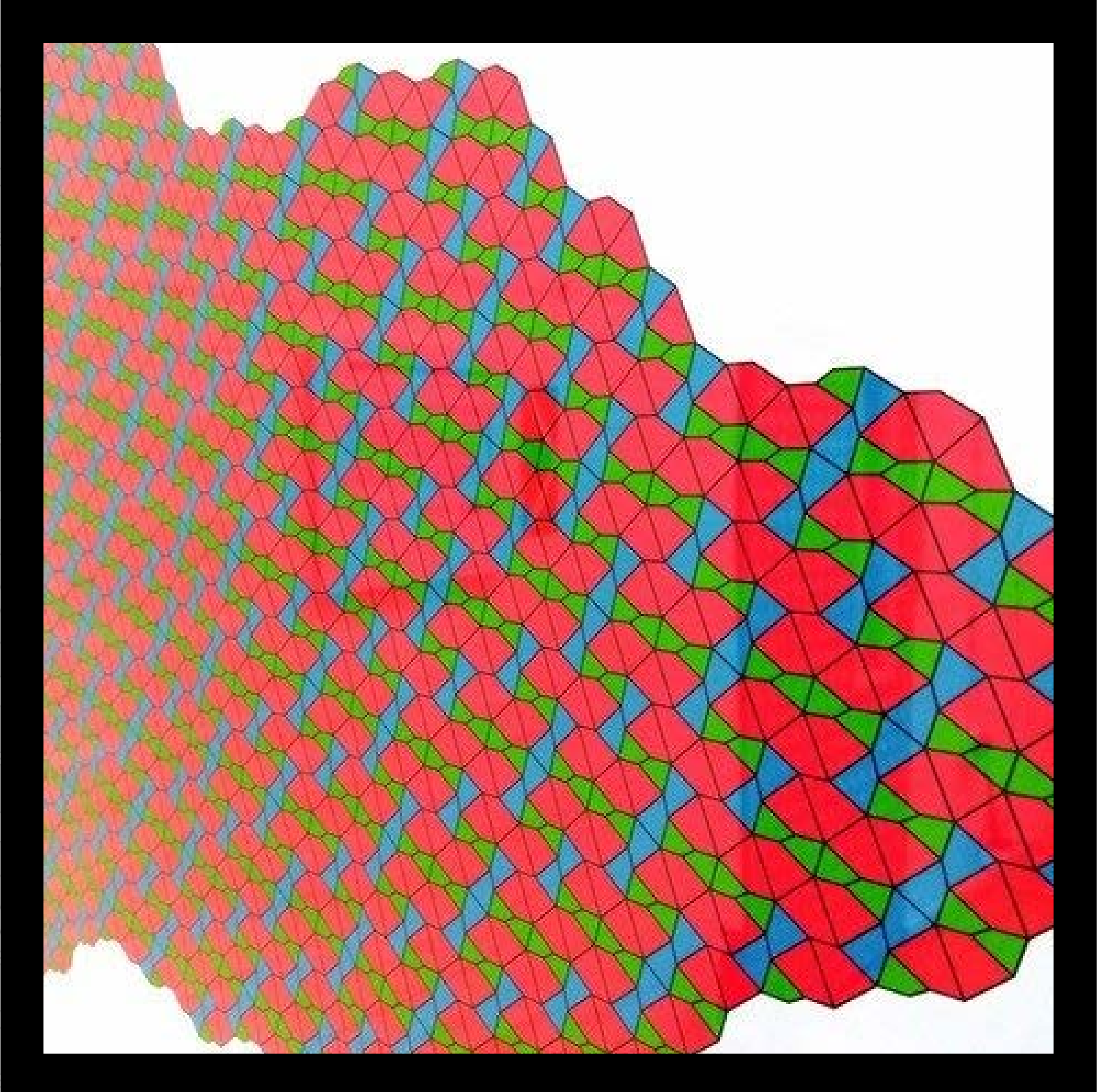
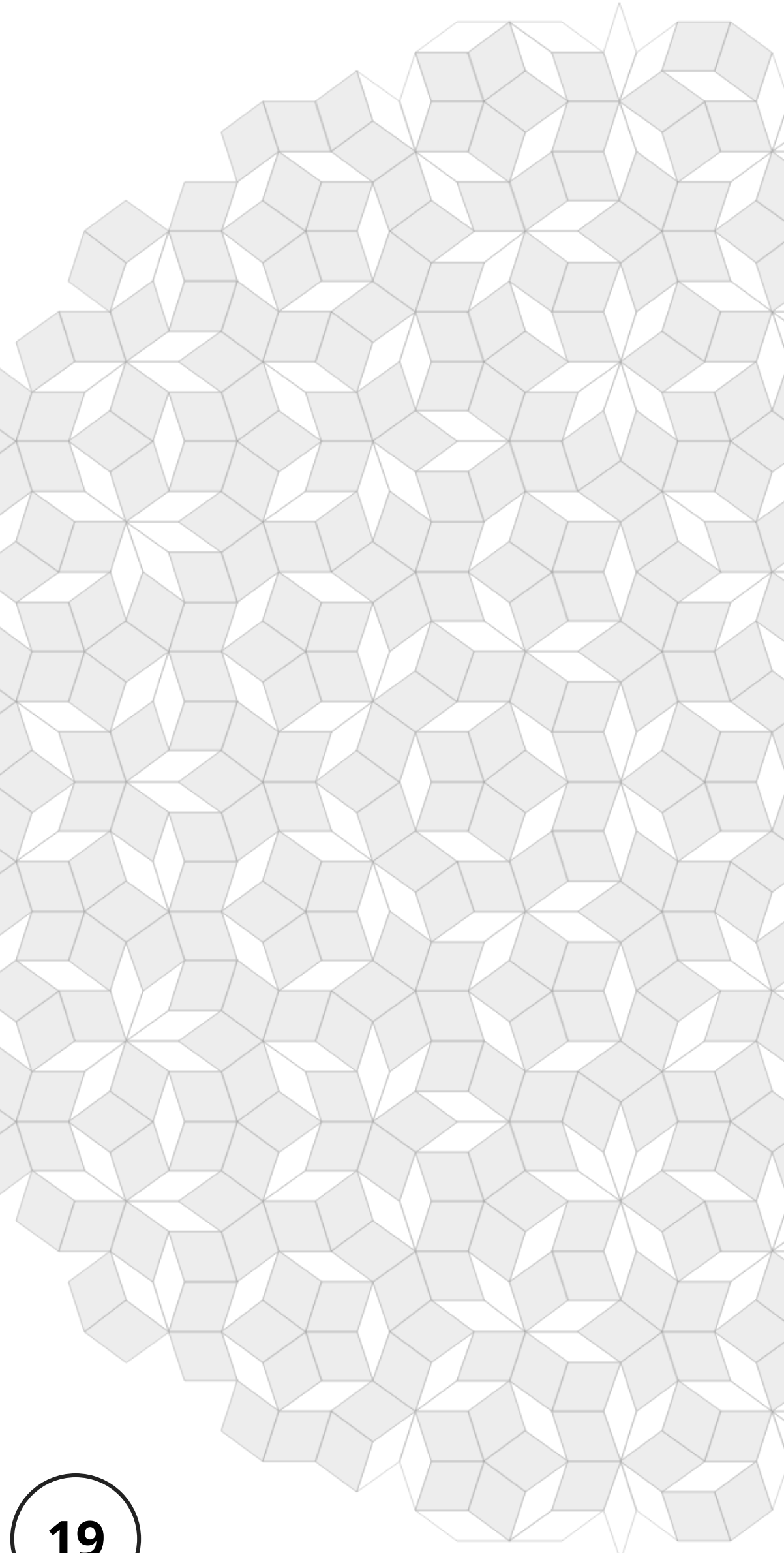


# LIAM TAYLOR-WEST



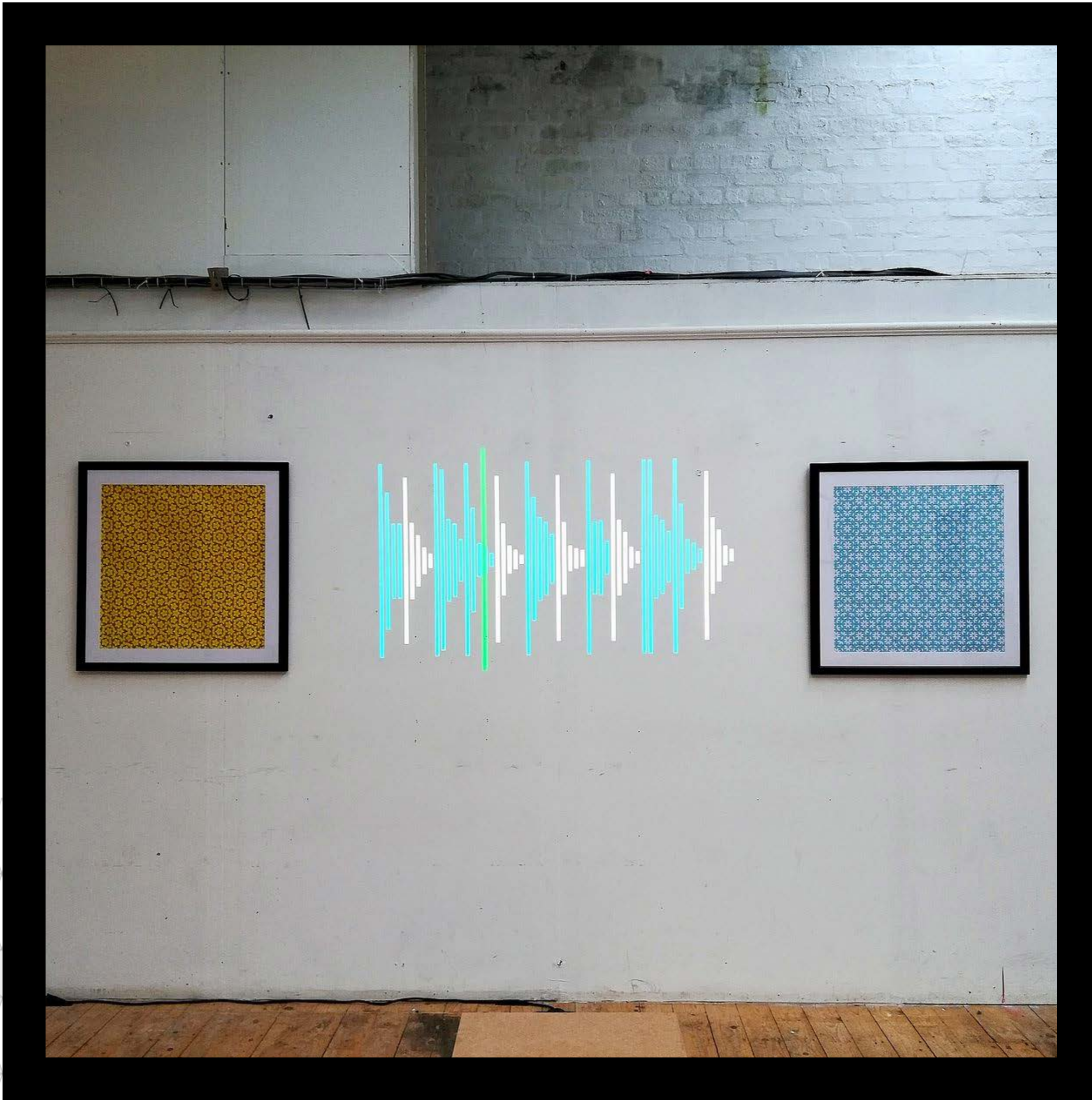
Liam is a Composer and Creator of Audiovisual Artworks. He was the recipient of the 2018 Ivors Composer Award in the Community or Educational Project category for The Umbrella, and has had his music performed by ensembles such as the City of Birmingham Symphony Orchestra, BBC Concert Orchestra, National Open Youth Orchestra and Bournemouth Symphony Orchestra. Liam is an advocate of the use of creative technology in composition and performance, and is a Resident at Watershed's Pervasive Media Studio, in Bristol.

[www.liamtaylorwest.com](http://www.liamtaylorwest.com)  
Twitter and Instagram:  
@liamtaylorwest

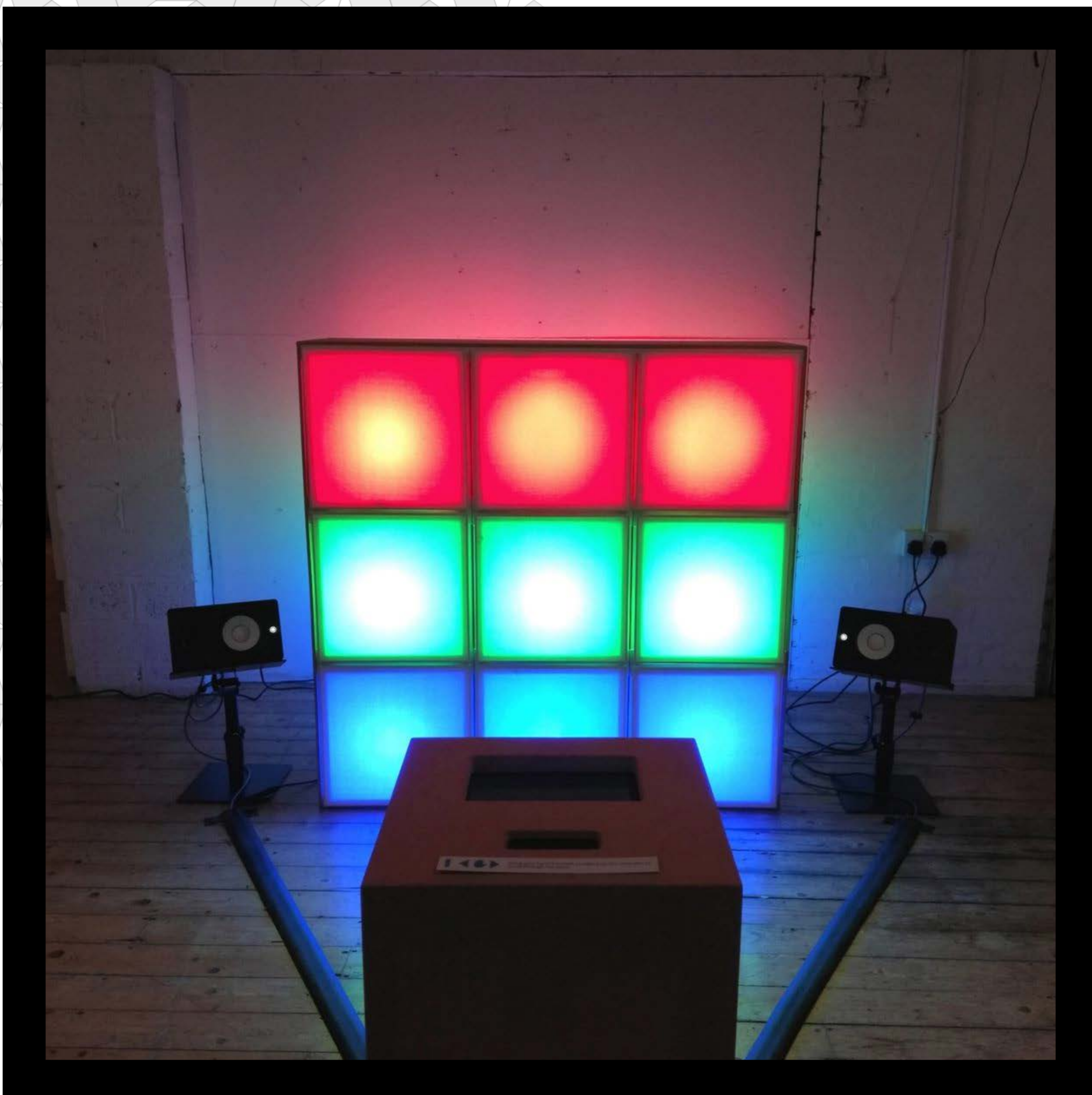


**Liam Taylor-West**





**Liam Taylor-West**



**Liam Taylor-West**



# MARYAM SMIT

Maryam Smit, is an experienced traditional and grassroots community based artist based in Milton Keynes, UK.

'I have always developed a strong interest for patterns, after travelling through Europe, and was particularly intrigued by the geometric compositions and structures in Islamic art. My passion to discover the beauty of Islamic geometric art has led me to share my experiences with MKIAC, developing new areas of design and expression through art. I am mostly self taught and have attended the Prince's School of Traditional Arts'.

Maryam particularly likes to use a range of different materials, creating beautiful contemporary and traditional artwork, educating and sharing her love for Islamic art patterns. She also has led the MKIAC outreach programme including schools, and her work has been exhibited within Buckinghamshire.

She was born and educated in The Netherlands where she studied Fashion Design and Business.

[www.mkiac.org](http://www.mkiac.org),

Instagram: @wearemkiac

Facebook: MKIAC

Twitter: @MKIAC



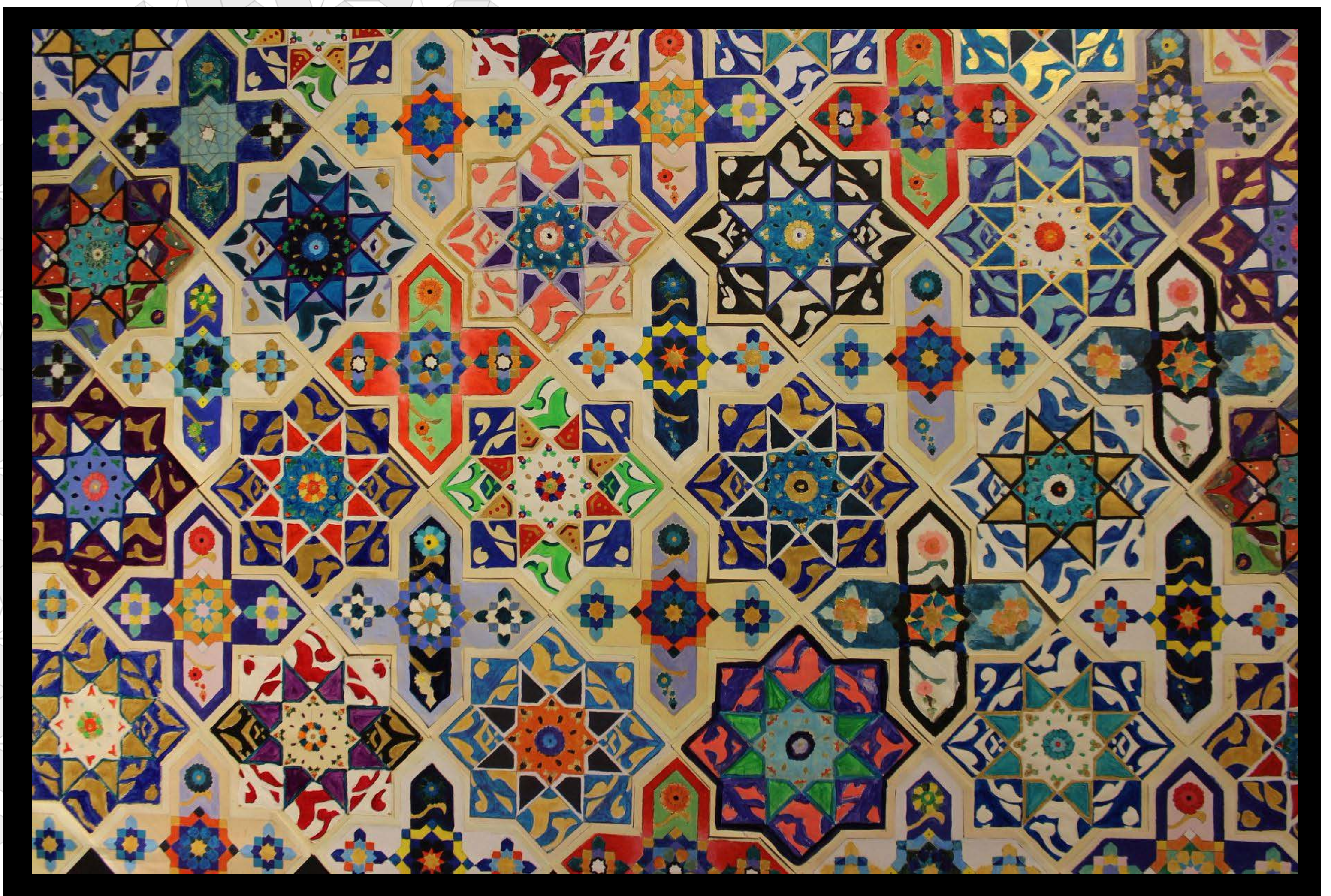




Maryam Smit

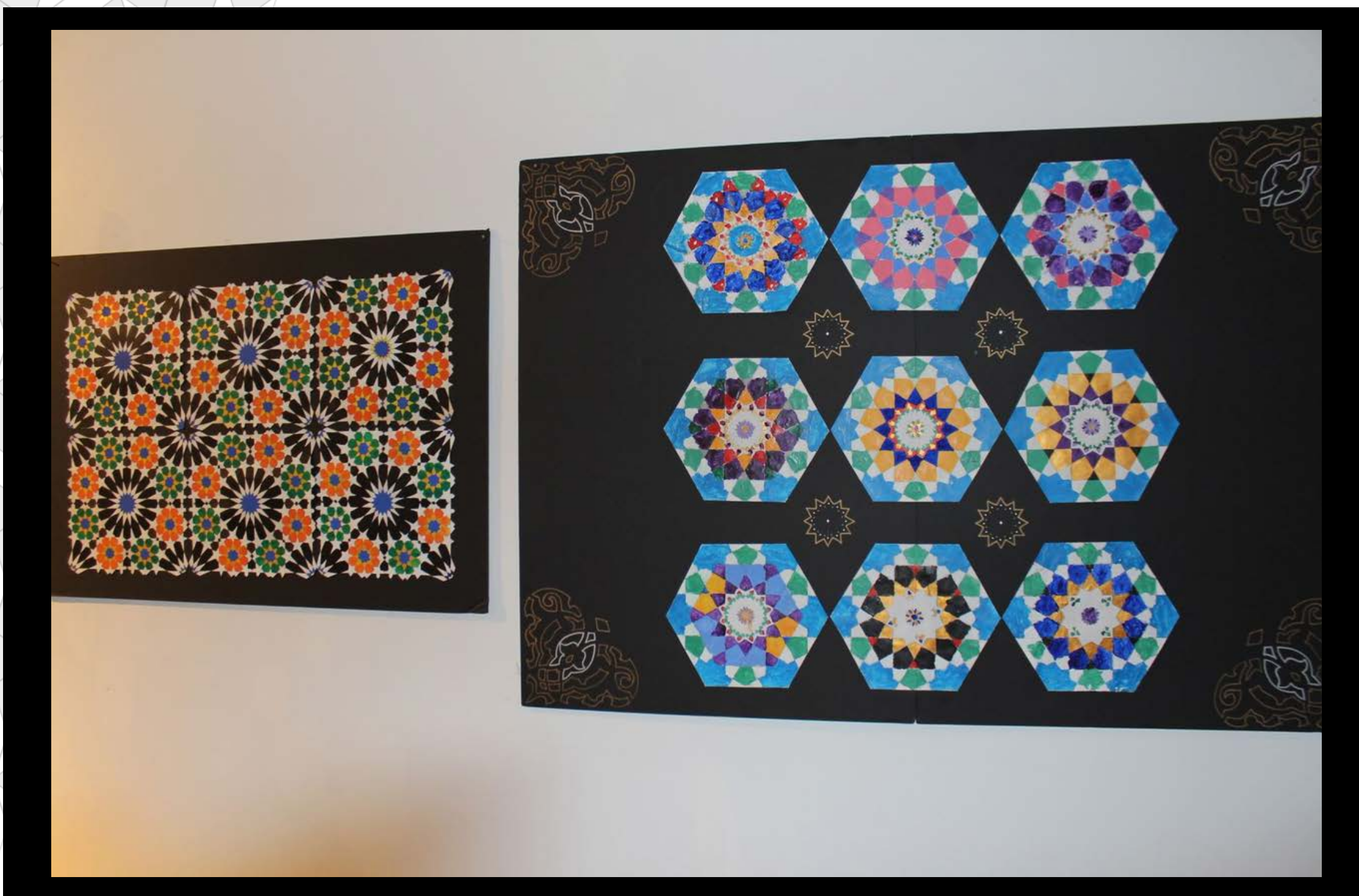
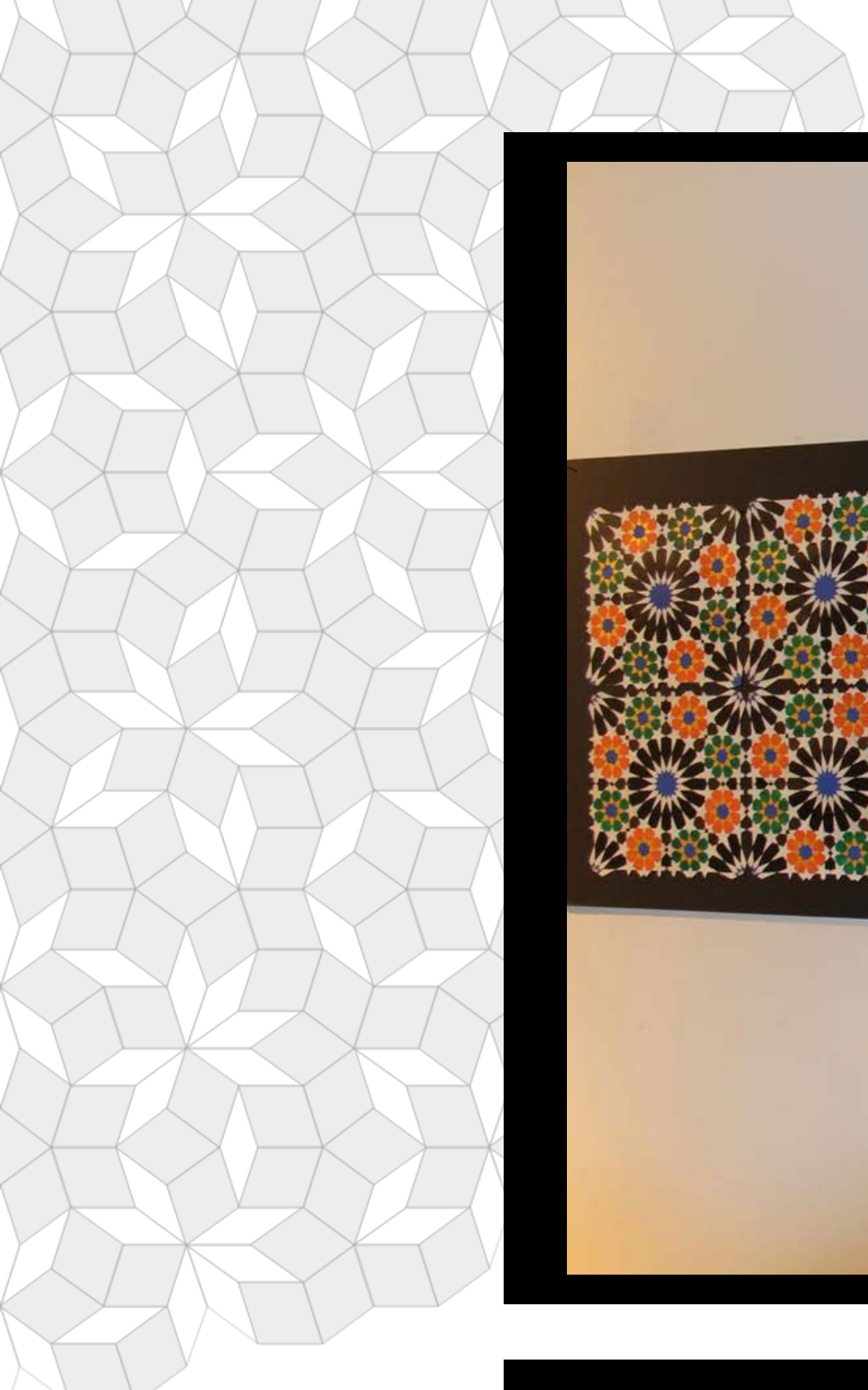


Maryam Smit



Maryam Smit





Maryam Smit



Maryam Smit



Maryam Smit





# NATALIE PRIEBE FRANK



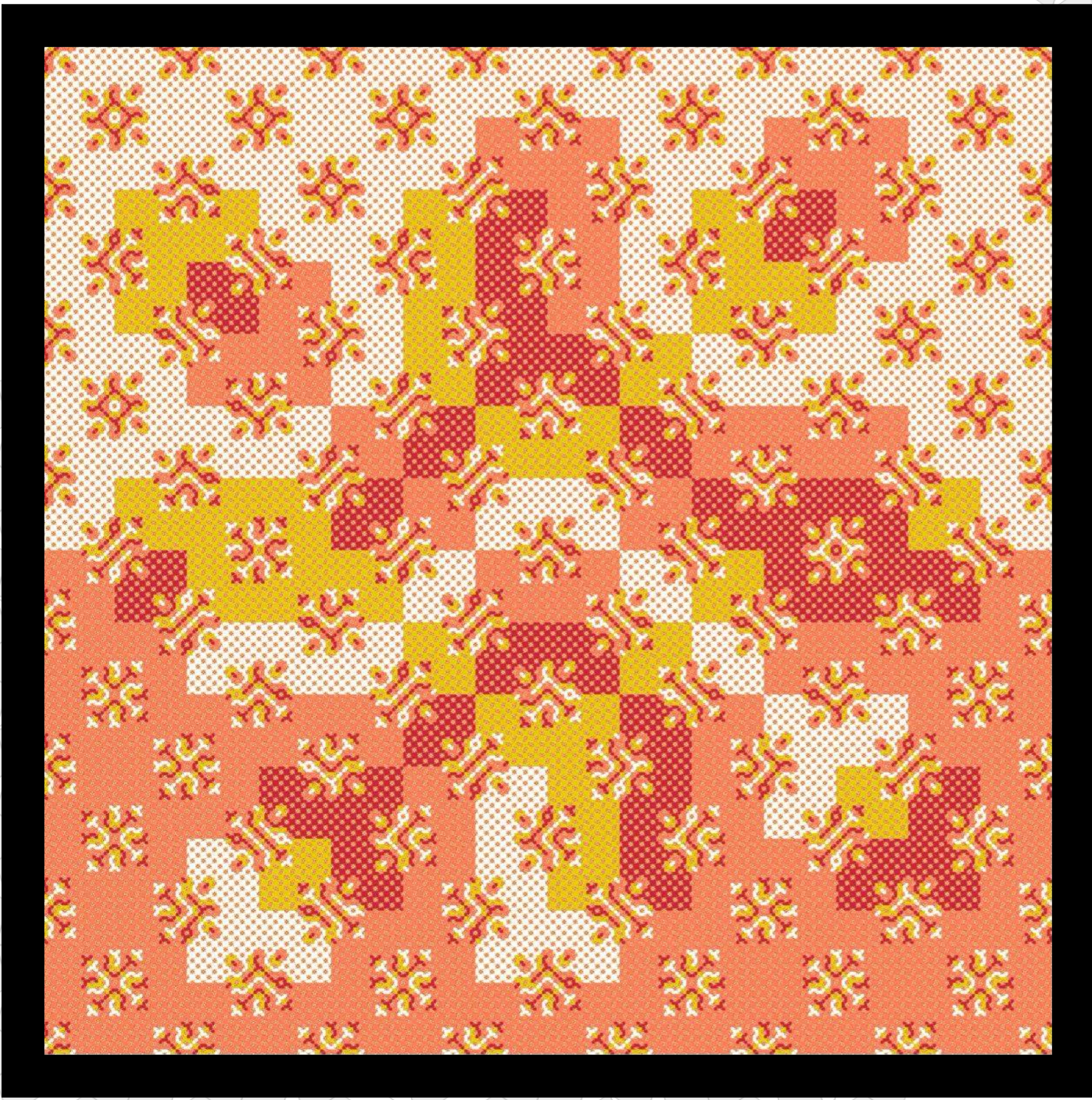
nafrank@vassar.edu

I am a mathematician who studies self-similar tilings and other models of aperiodic order. I have been making art based on my research for two decades. Scientific interest in the tessellations I study is due to the complexity of their structure, and this complexity is also the source of their artistic interest. Most of the art I produce features some form of self-similarity related to that found in fractals, except on an infinitely large rather than an infinitesimal scale.



Natalie Priebe Frank





**Natalie Priebe Frank**



**Natalie Priebe Frank**

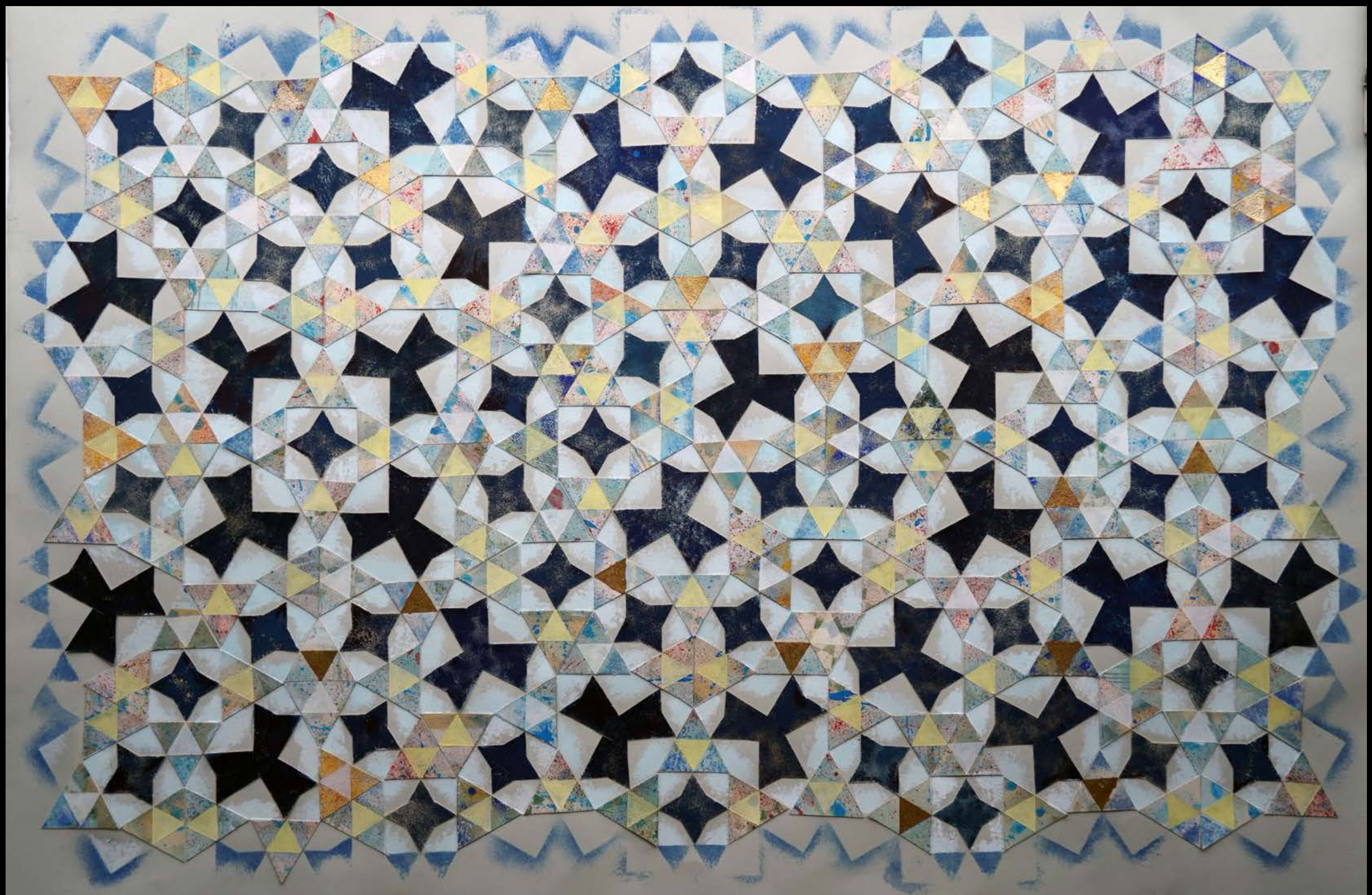


# RICHARD HENRY



richard\_henry@yahoo.com  
<http://www.richardhenry.art>  
Instagram and Facebook:  
@richardhenryart

Richard Henry is an artist with a particular interest in Islamic geometric patterns. He has taught widely on this topic, including developing courses for the British Museum in London. He regularly leads art tours to the Islamic world to explore patterns in context and has carried out research in Morocco, Egypt, Syria, Turkey and Iran. He is interested in traditional techniques of construction by hand, as well as methods for innovating new designs.



**Richard Henry**



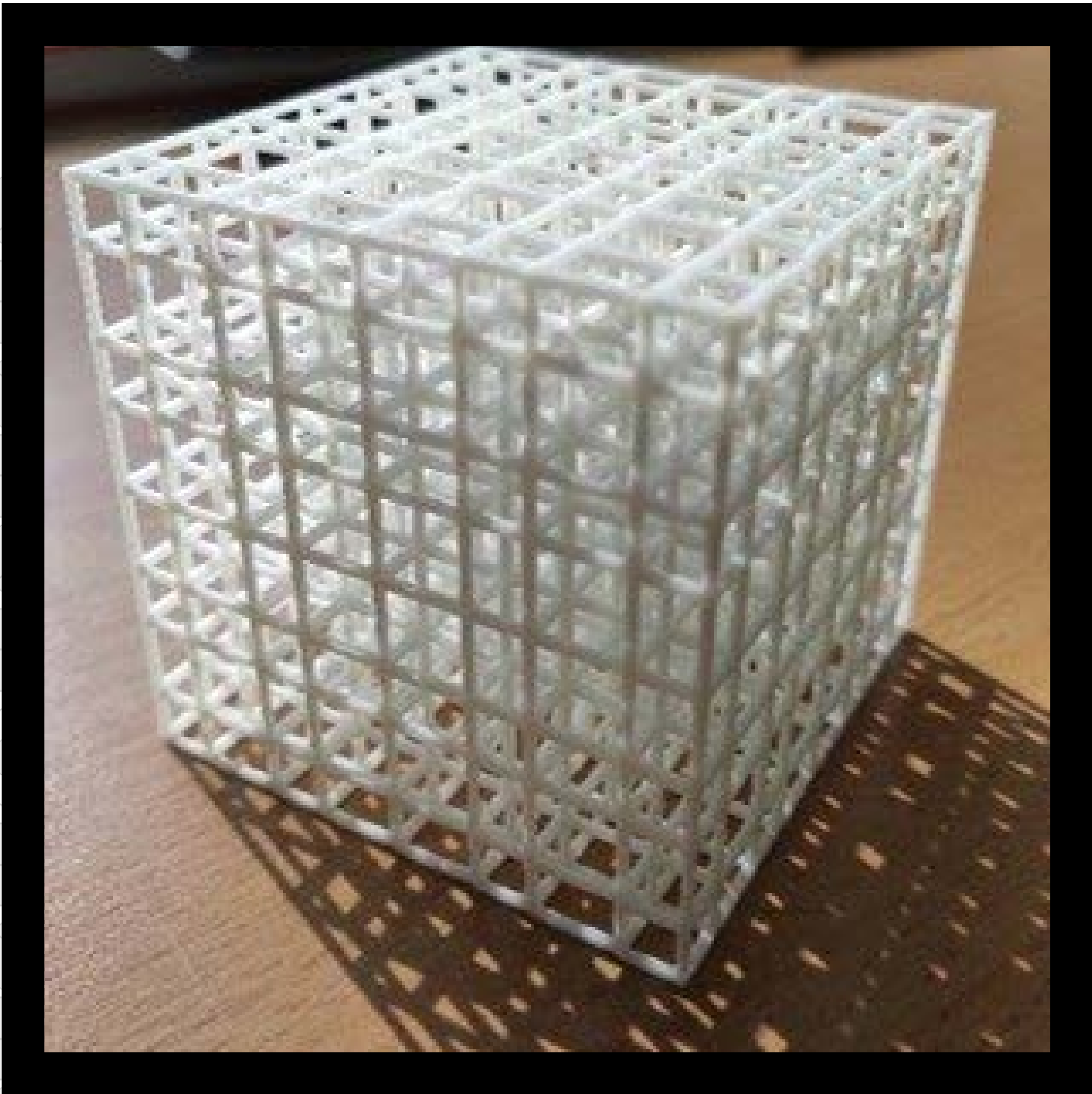


**Richard Henry**



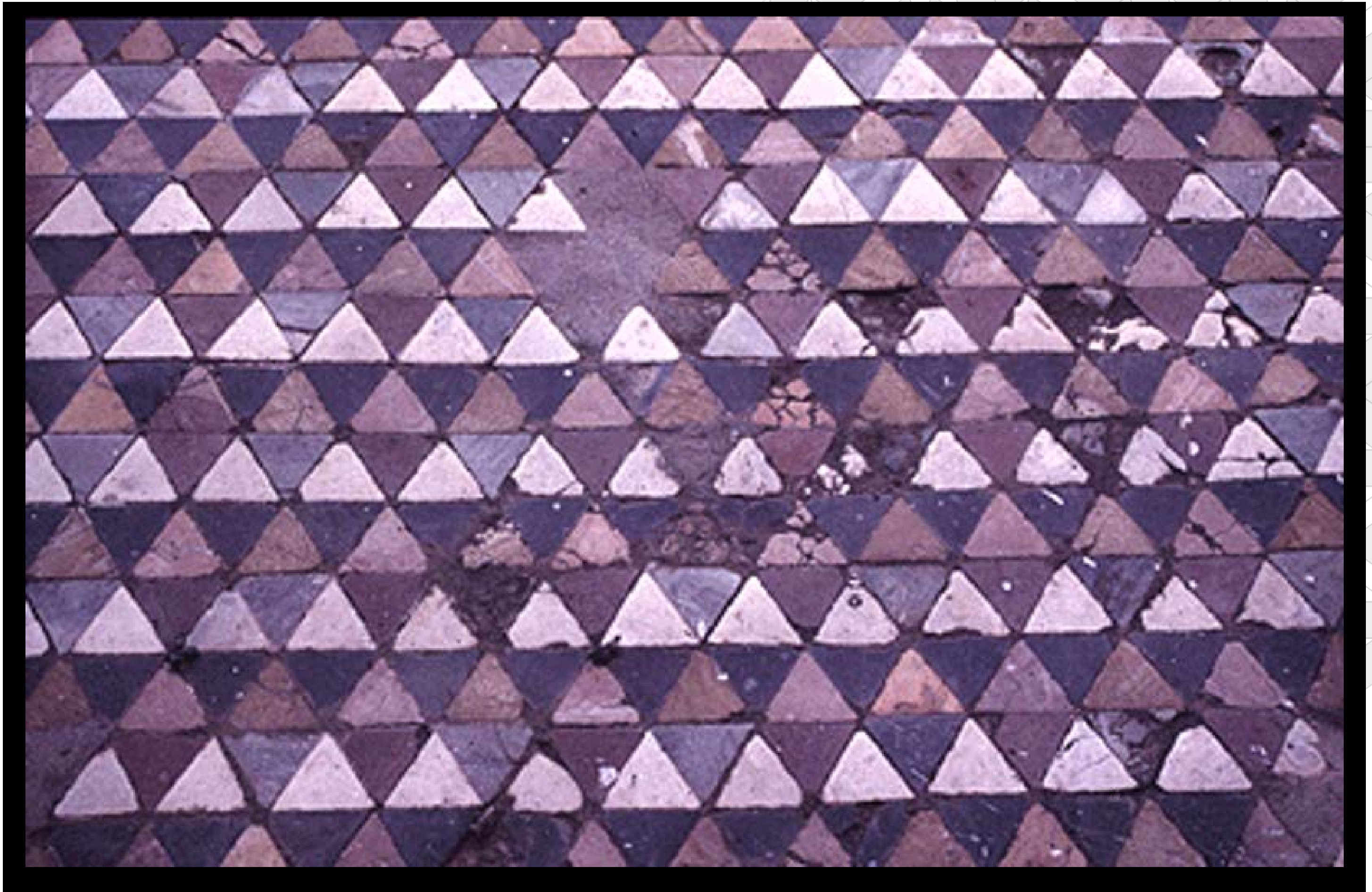
# RONAN MCGRATH

Ronan McGrath studied physics in Trinity College Dublin. He was appointed to a lectureship in physics at the University of Liverpool in 1990 and awarded a personal chair in 2004, with research interests in quasicrystals. He co-chaired Aperiodic 09 with Uwe Grimm.



**Ronan McGrath**





**Ronan McGrath**



**Ronan McGrath**



# ROSE VICKERS

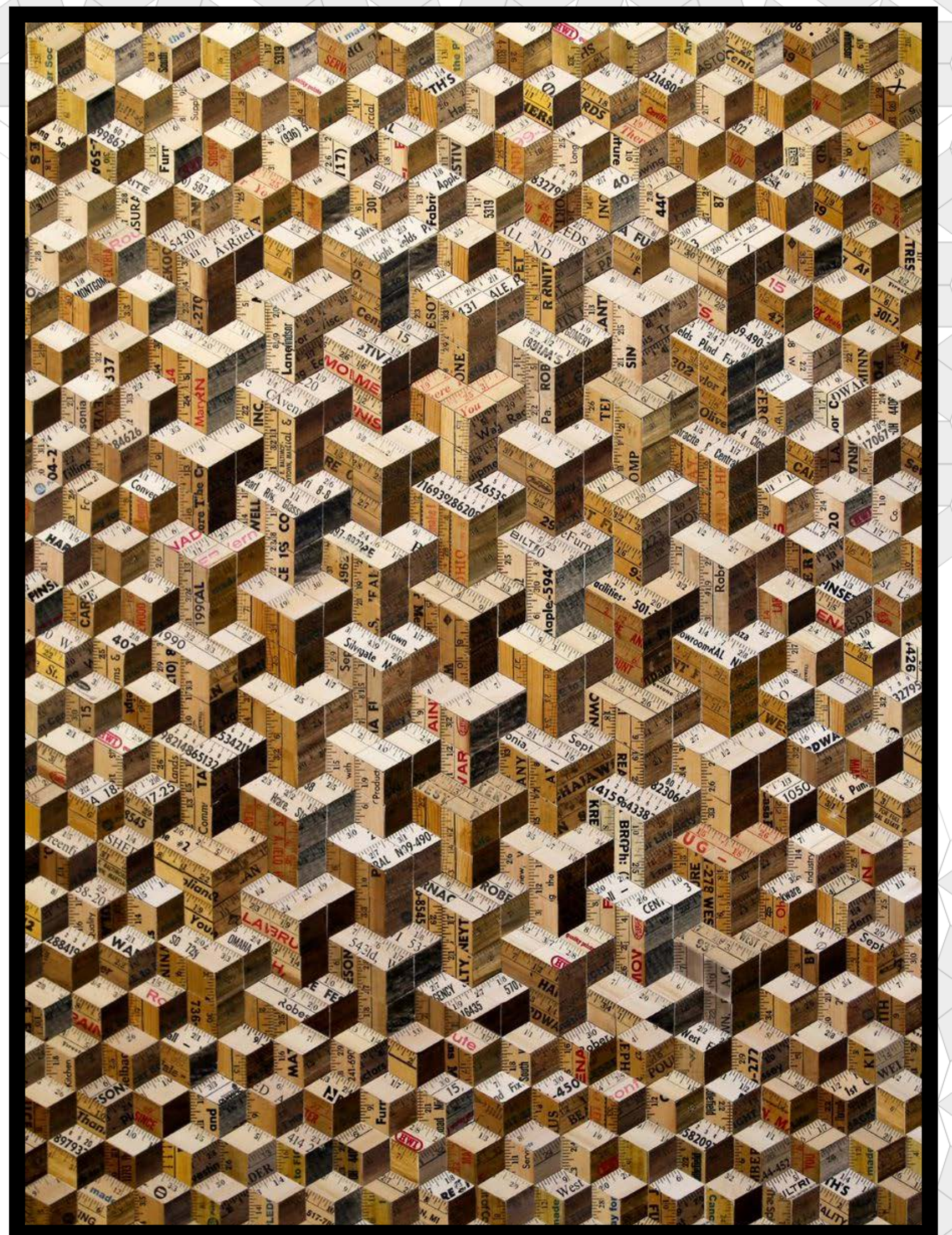
I love old rulers and I've been collecting them for years.

I especially like the marks and patination they acquire over time – years of being held by small fingers laboriously underlining titles and measuring things in maths.

The work I make is a response to this most commonplace of objects and I often use an untutored and almost instinctive geometry in the things I make, employing methods similar to (and suggestive of) the piecing of patchwork quilts.

I have exhibited widely throughout UK and abroad and have pieces in private and public collections in Europe and the United States.

[www.rosevickers.co.uk/about](http://www.rosevickers.co.uk/about)





# SAMIRA MIAN

Samira Mian uses her Mathematics and Education background to share her knowledge and artistic passion. Breathing new life into the centuries-old technique of straight edge & compasses geometry to recreate geometric patterns from the Islamic Lands. Expressing her creativity by creating artwork in a contemporary style using colourful watercolours and metallic paints.

She regularly leads workshops as well as reaching many online through Zoom classes, social media and her YouTube channel. Contextualising the patterns so that we can enjoy this beautiful intersection of history, religion, geography, art, design and mathematics that occurs when we recreate these beautiful geometric patterns



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Samira Mian





**Samira Mian**



**Samira Mian**

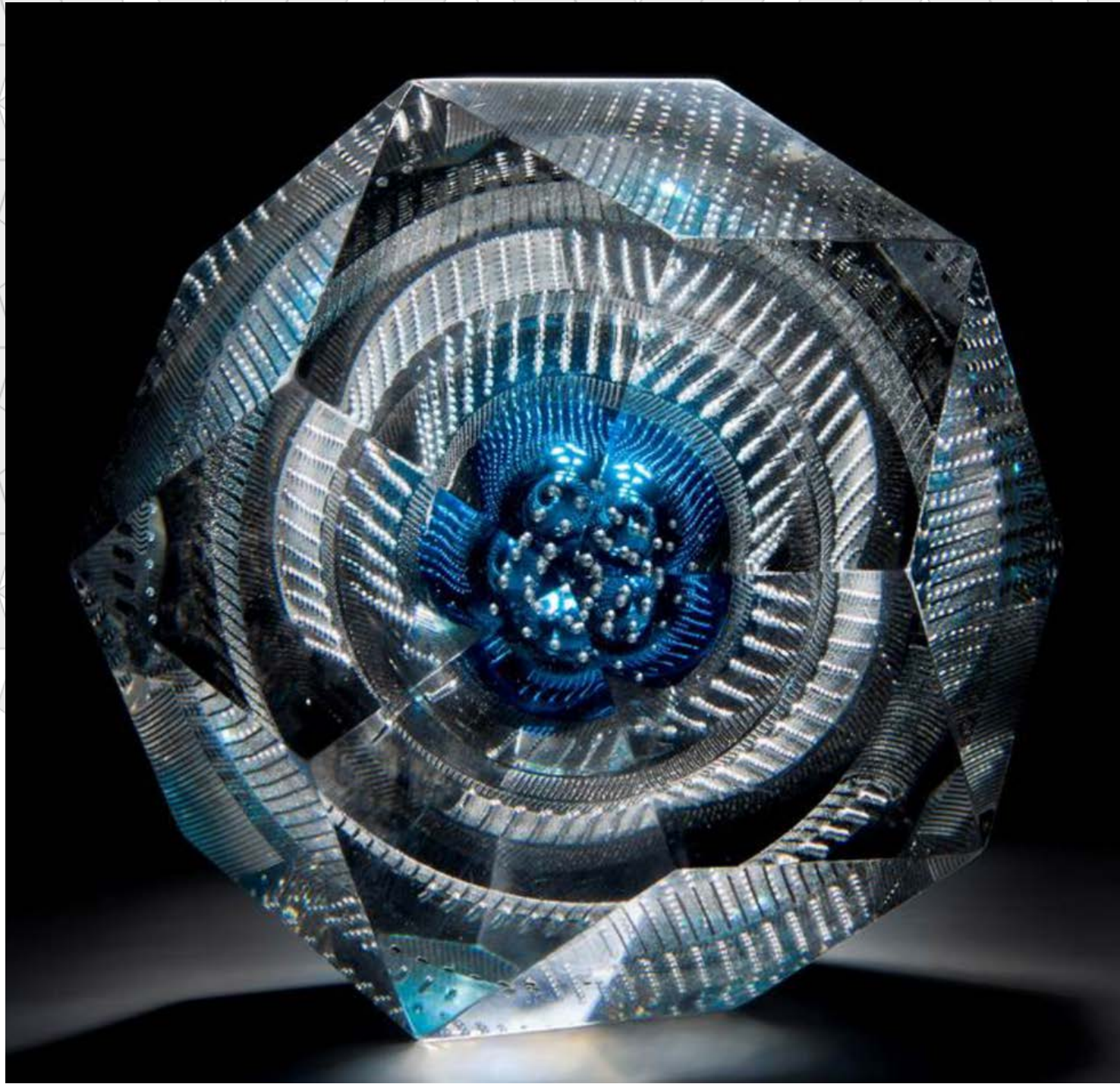


# SHELLEY JAMES

Dr Shelley James is a TedX speaker, glass artist, lighting designer and a qualified electrician. She is Artist in Residence at the Bristol Eye Hospital, elected member of the Royal Society of Sculptors and curates the International Day of Light Festival with the London Institute for Nanotechnology.



Photo Credit to Marcus Ahmad



Shelley James & Brian Sutton



Shelley James & Brian Sutton



Shelley James & Brian Sutton





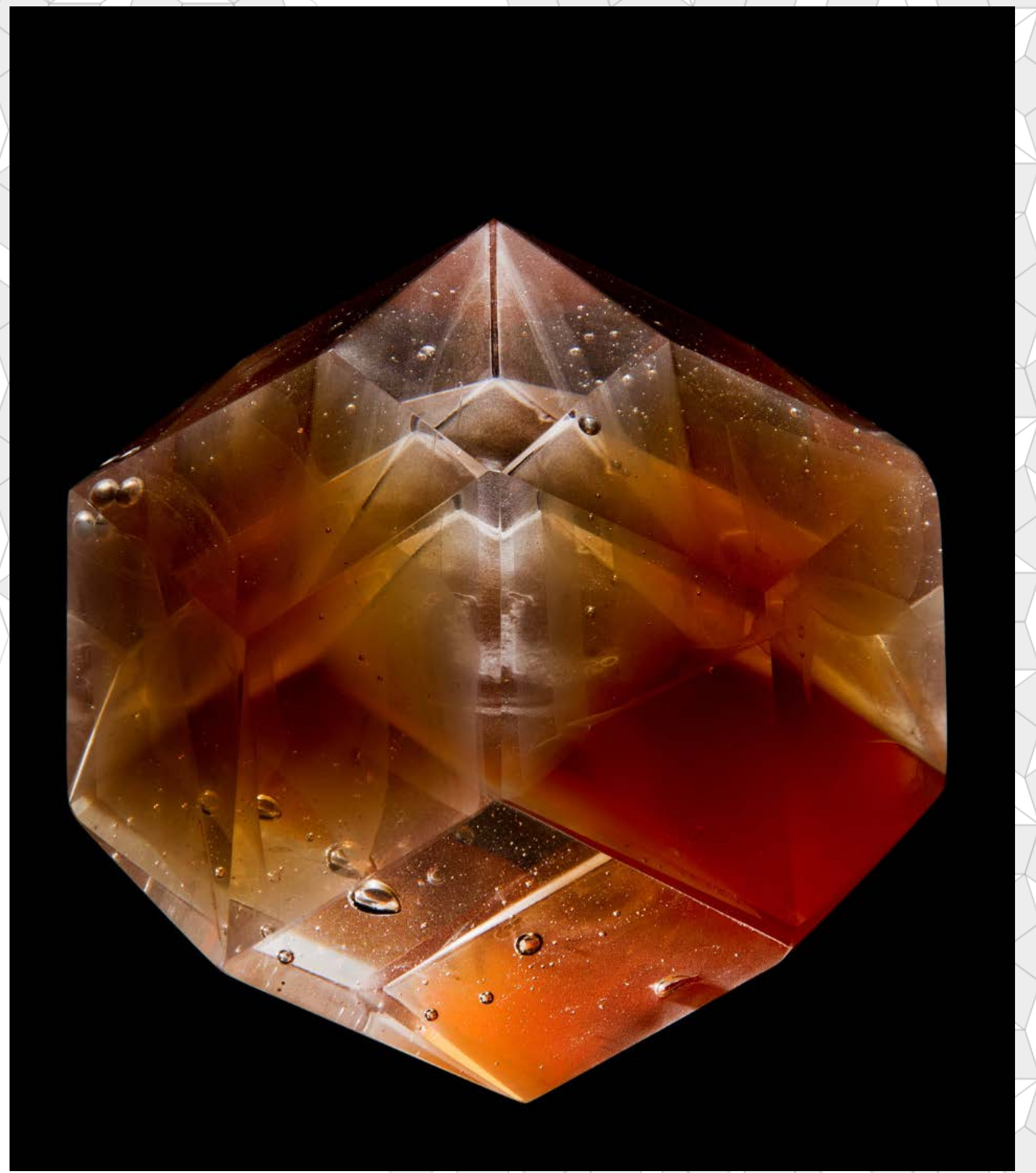
**Shelley James & Brian Sutton**



**Shelley James & Brian Sutton**



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**Shelley James & Brian Sutton**



**Shelley James & Brian Sutton**



# BRIAN SUTTON

Brian Sutton is Professor of Molecular Biophysics at King's College London. He studied chemistry at Oxford University and then trained in X-ray crystallography, holding a Royal Society University Research Fellowship in Oxford before moving to King's. His research uses X-ray crystallography to determine the atomic structures of antibodies involved in allergic diseases, in particular asthma, in order to develop new therapeutic agents. Brian has collaborated with Shelley James on exhibitions such as *Crystal Symmetries* (History of Science Museum, Oxford) and *Dynamic Symmetry* (The Royal Society), exploring both classical symmetry, and more recently quasi-symmetry and aperiodic tiling, through public engagement activities.

[brian.sutton@kcl.ac.uk](mailto:brian.sutton@kcl.ac.uk)

All artworks are in collaboration with Shelley James.

[brian.sutton@kcl.ac.uk](mailto:brian.sutton@kcl.ac.uk)





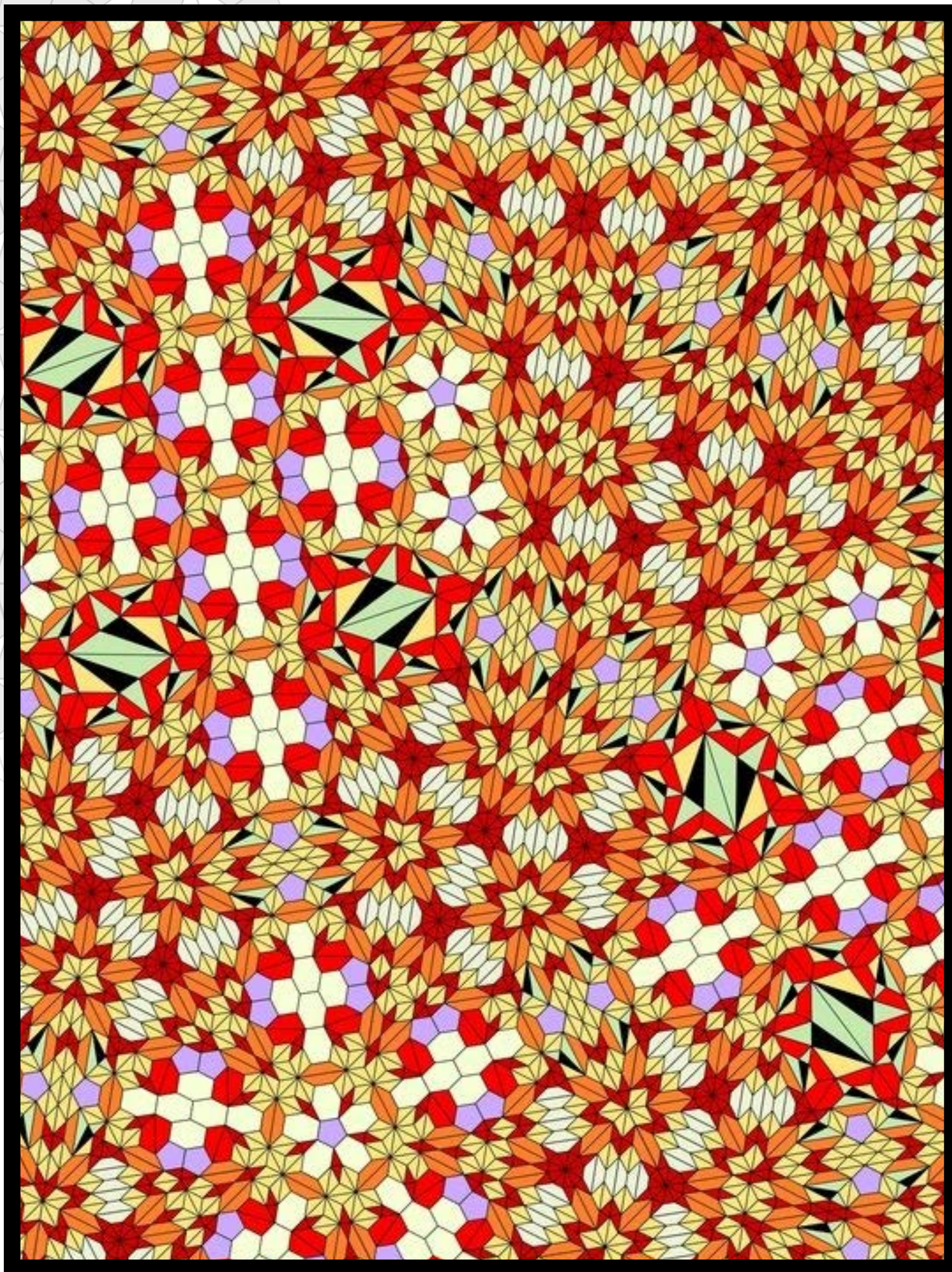
# STEFAN PAUTZE



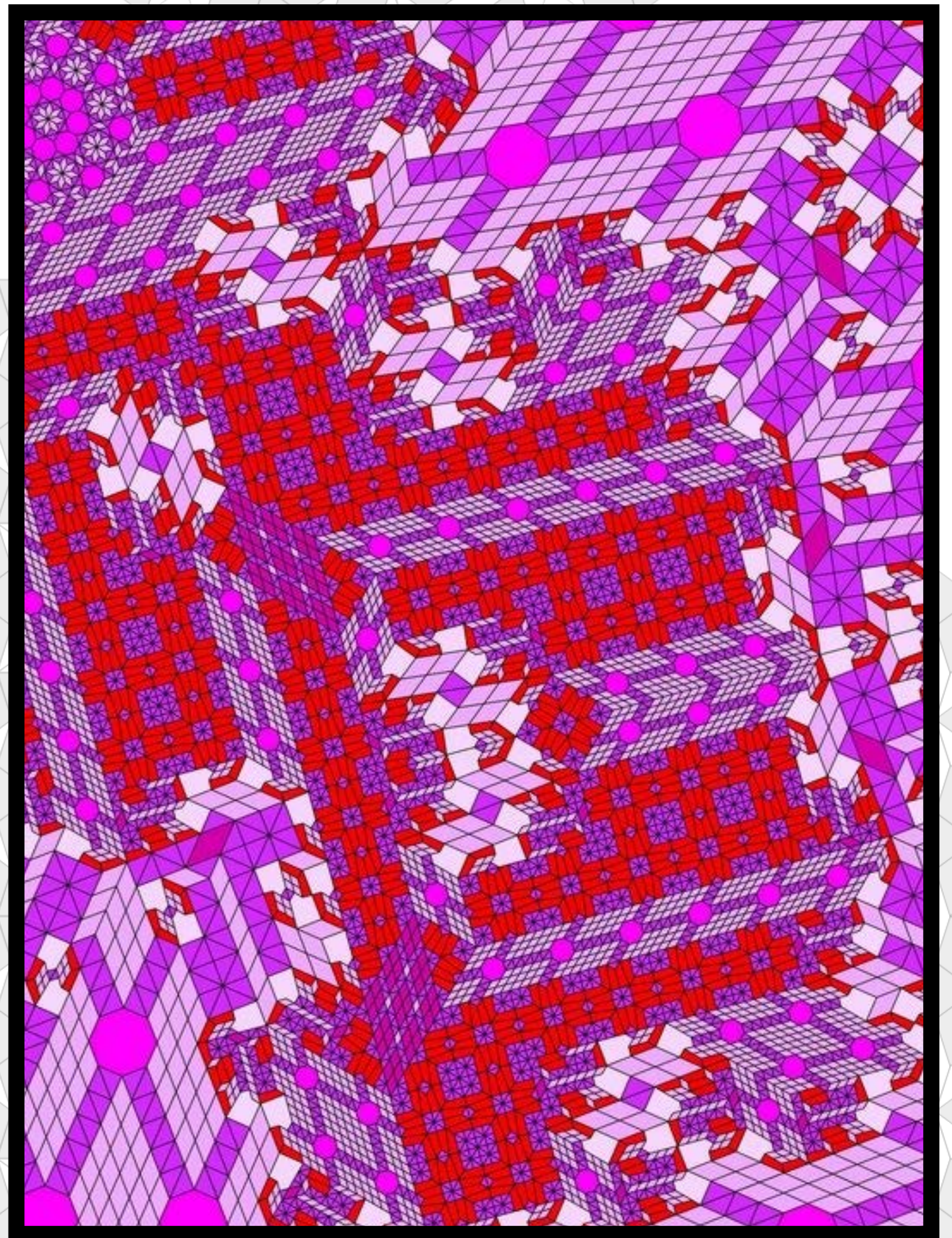
Stefan Pautze is graduate engineer in the automotive industry and graphic artist with great interest in aperiodic geometry and the Demoscene. He lives and works in Pörnbach, Germany.

[stefan@pautze.de](mailto:stefan@pautze.de)

<http://pautze.de>

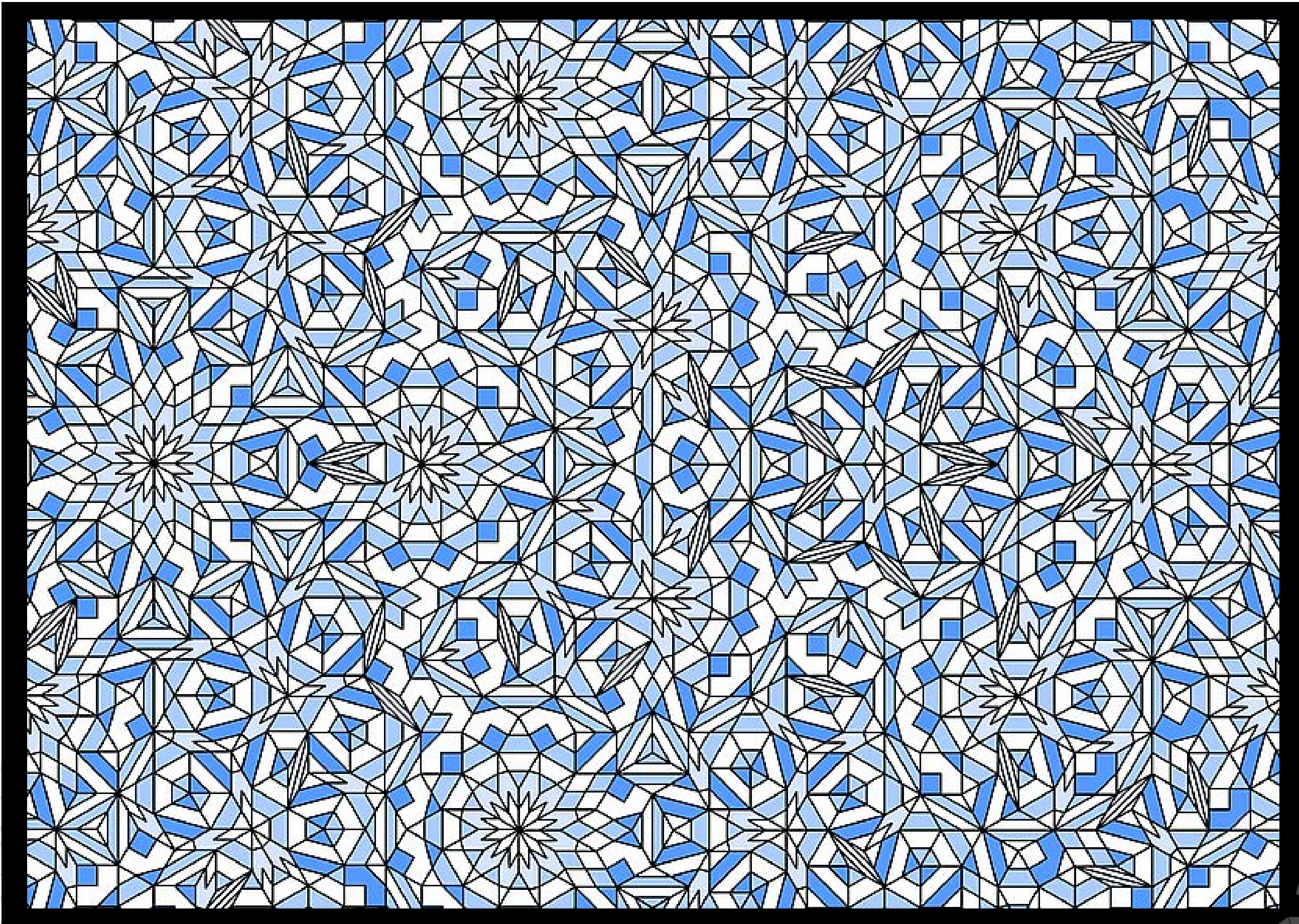


**Stefan Pautze**

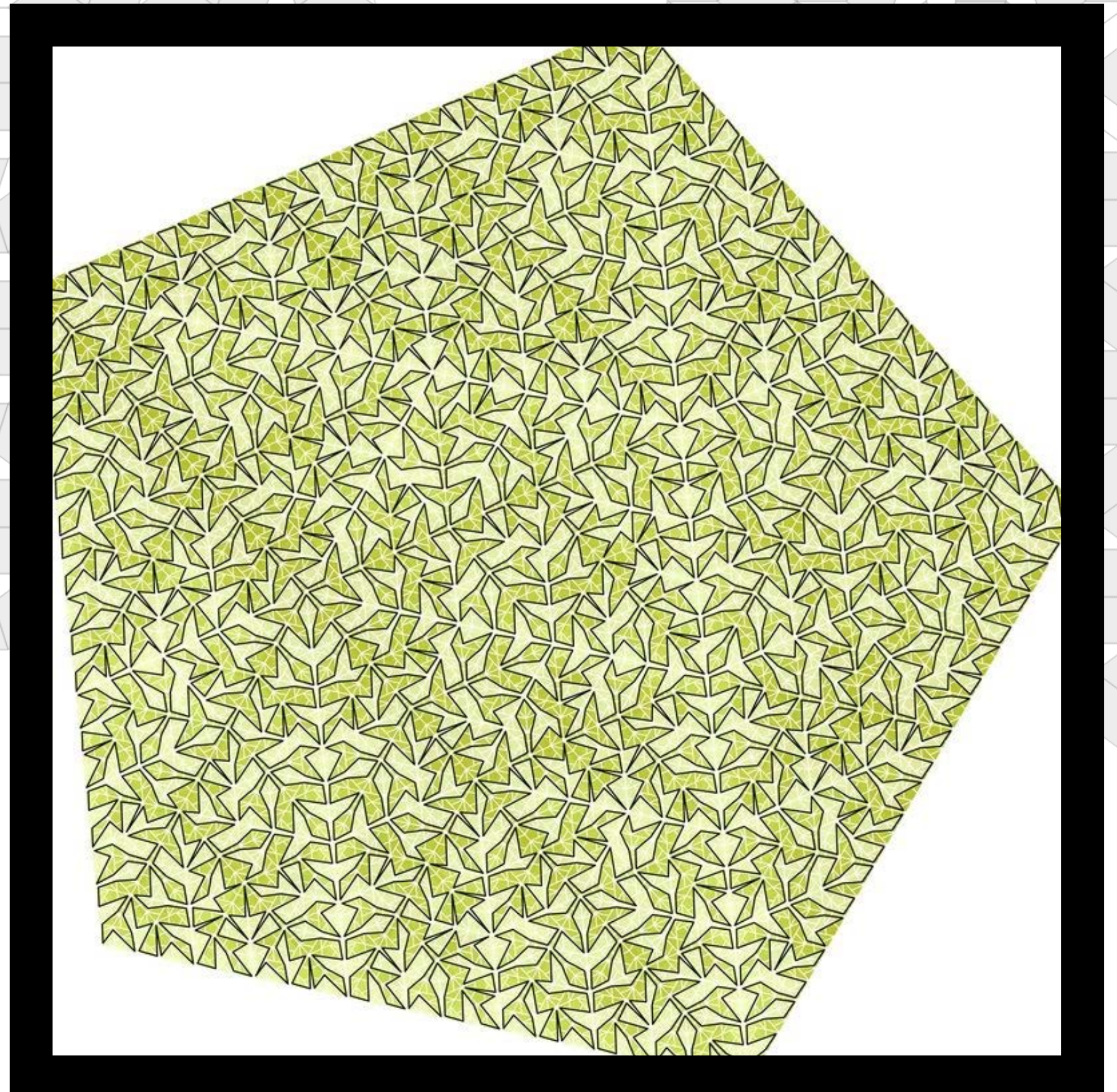


**Stefan Pautze**



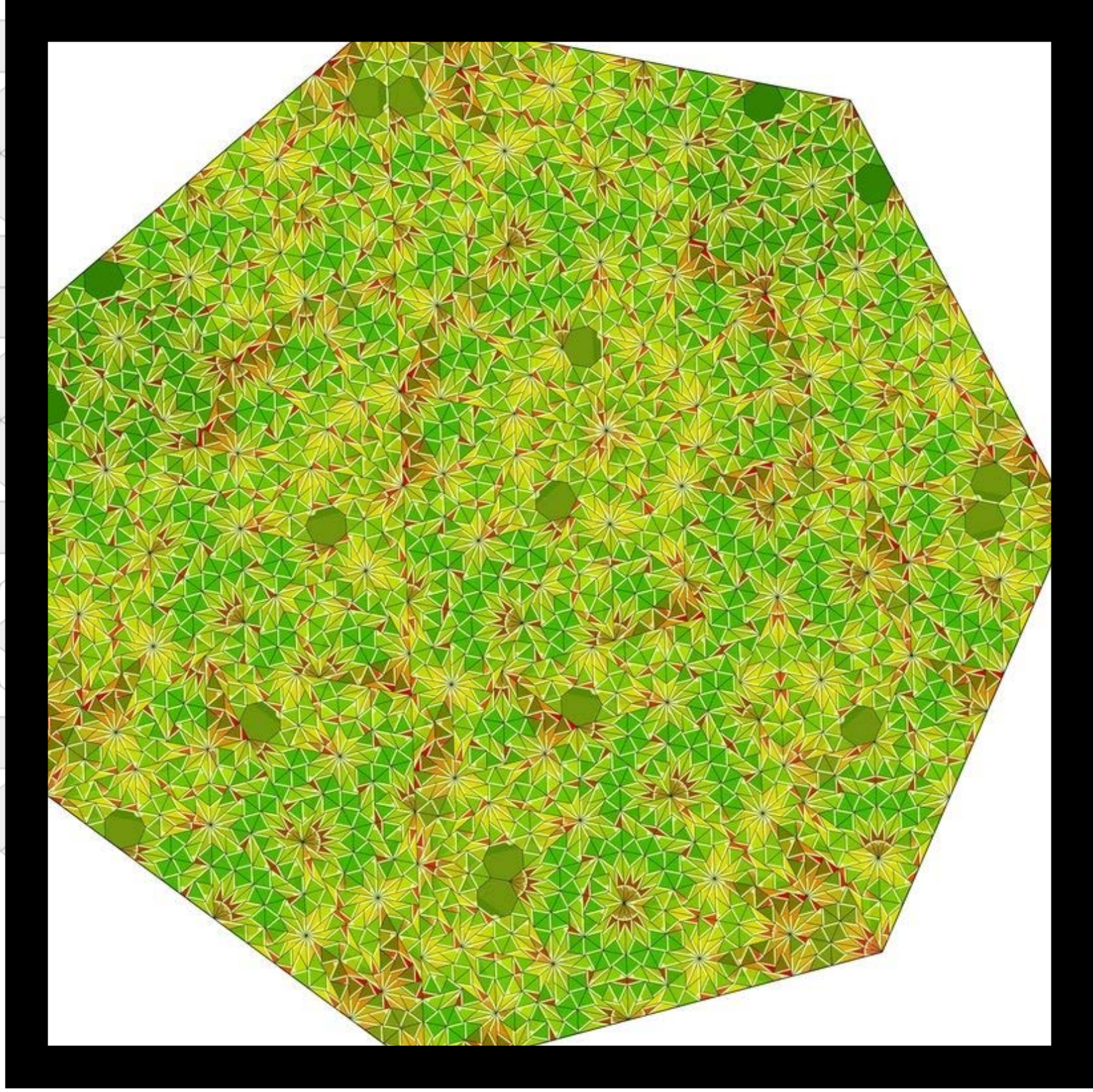


**Stefan Pautze**

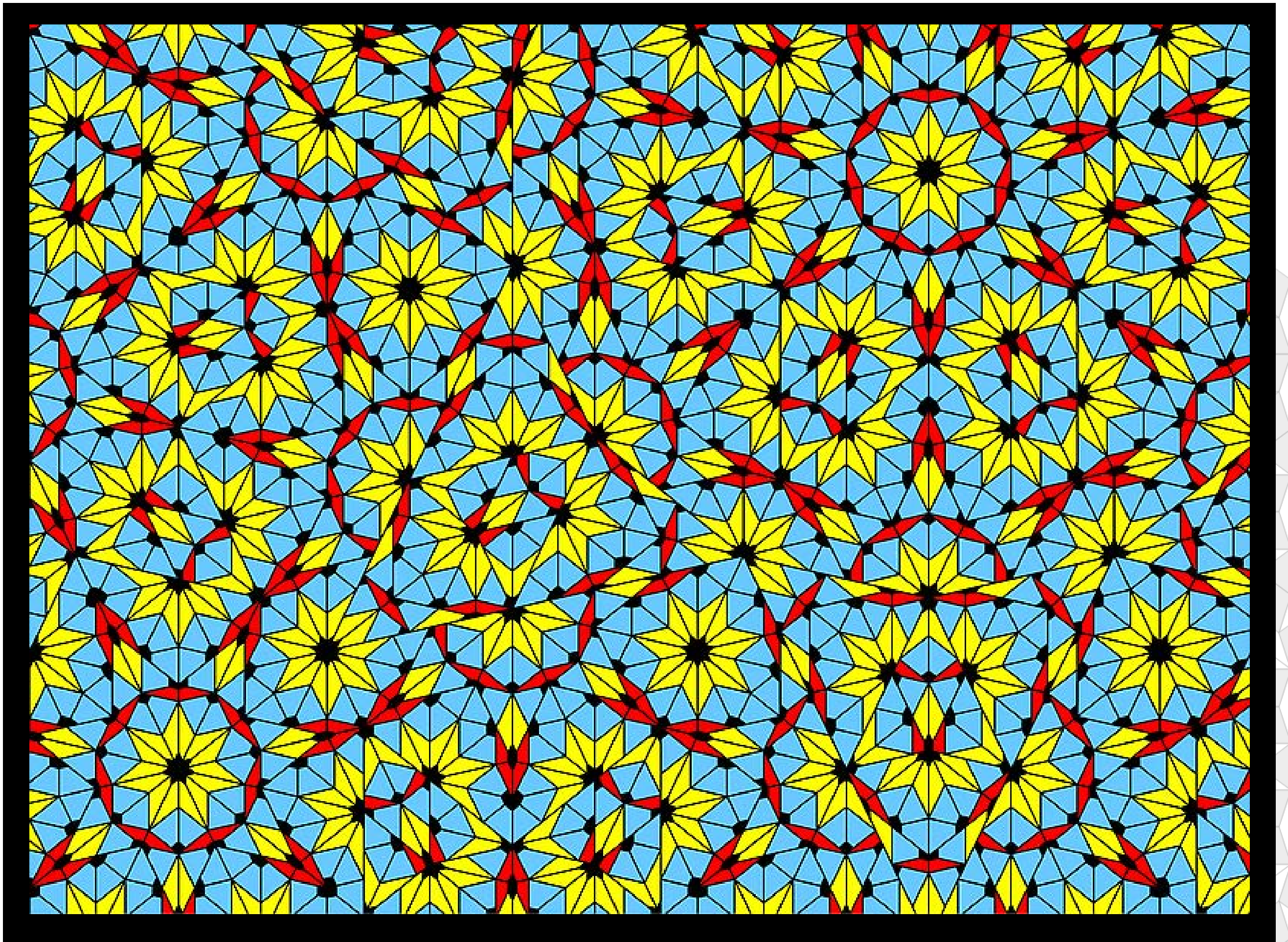


**Stefan Pautze**





**Stefan Pautze**



**Stefan Pautze**







