

Perceptual maps: the good, the bad and the ugly

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Abstract

Perceptual maps are often used in marketing to visually study relations between two or more attributes. However, in many perceptual maps published in the recent literature it remains unclear what is being shown and how the relations between the points in the map can be interpreted or even what a point represents. The term perceptual map refers to plots obtained by a series of different techniques, such as principal component analysis, (multiple) correspondence analysis, and multidimensional scaling, each needing specific requirements for producing the map and interpreting it. Some of the major flaws of published perceptual maps are omission of reference to the techniques that produced the map, non-unit shape parameters for the map, and unclear labelling of the points. The aim of this paper is to provide clear guidelines for producing these maps so that they are indeed useful and simple aids for the reader. To facilitate this, we suggest a small set of simple icons that indicate the rules for correctly interpreting the map. We present several examples, point out flaws and show how to produce better maps.

Keywords: Perceptual map, correspondence analysis, multiple correspondence analysis, principal component analysis, multidimensional scaling, biplot.

1 Introduction

The increase of computer efficiency and memory capacity has led to an enormous expansion of data collection. Consequently, businesses and organisations possess large quantities of data. These data contain important information covering several important areas. However, the size and complexity of the data sets require smart and efficient ways of distilling relevant information. Visualisation of complex data may help to separate the information from the “chaff” by allowing us to use our natural spatial/visual abilities to identify patterns in complex multivariate data and/or to determine where further exploration should be done (Tegarden, 1999; Jarvenpaa, 1989; DeSarbo et al., 2001). Furthermore, human abilities to recognise, process and remember visual patterns make visualisations particularly suited to communicate the essence of the data (Spence and Lewandowski, 1990).

The versatility and power of graphical representations of complex high dimensional data has long been acknowledged in marketing research (Frank and Green, 1968; Steffire, 1969; Green and Carmone, 1969, 1970) and practice (Doehlert, 1968; Huber, 2008). There are numerous applications in which graphical representations are used to assist marketing managers with issues related to market segmentation (Natter et al., 2008; DeSarbo et al., 2008; DeSarbo and Wu, 2001), market structure analysis (Wind, 1982; Katahira, 1990; MacKay and Zinnes, 1986), product design and positioning (DeSarbo and Rao, 1986; Eliashberg and Manrai, 1992; Shocker and Srinivasan, 1974; Pessemier and Root, 1973), brand-switching (DeSarbo and Manrai, 1992; Dillon and Gupta, 1996; Lehmann, 1972), customer value (Sinha and DeSarbo, 1998), customer satisfaction (Wu et al., 2006), and understanding relationships among consumer perceptions (Lee et al., 2002; Manrai and Sinha, 1989; Torres and Bijmolt, 2009). In most cases, the graphical representations serve as a diagnostic tool. However, attempts have been made to use them in a predictive setting (Pessemier and Root, 1973; Shocker and Srinivasan, 1974). There are several methods used to produce graphical representations for high dimensional data; in marketing the resulting data visualisations are often referred to as perceptual maps. Methodology yielding such perceptual maps is therefore simply referred to as “perceptual mapping”.

What actually is perceptual mapping? Hair et al. (1995, p.487) define a perceptual map as a “visual representation of a respondent’s perceptions of objects on two or more dimensions.” Lilien and Rangaswamy (2003, p.119) give as definition, a “graphical representation in which competing alternatives are plotted in Euclidean space”. Although initially used only in combination with multidimensional scaling methods (Frank and Green, 1968; Green and Carmone,

1969; Lehmann, 1972), the term now appears to be used in conjunction with any multivariate analysis method yielding a graphical representation of the data (Day et al., 1979), e.g. canonical discrimination (Pankhania et al., 2007), principal component analysis (Hartmann et al., 2005), correspondence analysis (Torres and van de Velden, 2007), and canonical correlation analysis (Taks and Scheerder, 2006). In addition, new methods yielding perceptual maps are developed for specific applications, (see, e.g., Chintagunta, 1998; Katahira, 1990; DeSarbo and Hoffman, 1987; DeSarbo and Manrai, 1992; DeSarbo and Young, 1997; DeSarbo et al., 2008; Sinha and DeSarbo, 1998).

The common element among these methods is that they can all be termed multidimensional analyses¹, to indicate that, potentially, results are available in many dimensions. Overwhelmingly, only two dimensions are exhibited, as this gives two-dimensional maps that can be shown on a sheet of paper or on a computer screen. Multidimensional analyses are widely used in many fields of application. It so happens that in Market Research we are often interested in people's perceptions of products, or relationships between pairs of products. Thus perceptual mapping refers to the type of data (perceptual) coupled with multidimensional methodology (mapping).

Regardless of what specific statistical method is used to produce the perceptual map, the purpose of the map is to display complex information in an engaging graphical manner. Clear perceptual maps powerfully add weight to assertions in accompanying text about relationships between and within (possibly latent) attributes. By avoiding difficult statistical concepts (e.g. p -values, confidence intervals, hypotheses testing etc.), and relying on the human ability to deal efficiently with graphical data, perceptual maps are very appealing to applied marketing researchers. Indeed such maps might be the primary means a reader uses to assess the message that the article is conveying. Maps tend to stand out from the page, and are used as part of the summary information given by some electronic journal databases. Thus it is crucial that perceptual maps are presented in such a way that the information within them can be quickly and correctly assimilated.

Unfortunately, as will be shown, the graphical presentation of perceptual maps both in the methodological literature and in applications is often defective. In a literature study covering recent academic publications we found many problems that either prohibit meaningful interpretations of perceptual maps or considerably complicate interpretations. We describe our findings in more detail later in the paper. The nature of the encountered problems is diverse. Sometimes, the problem is insufficient information given either, or both, in the caption or in the labelling of elements in the body of the map. Sometimes, it is misleading relative scaling of the

¹Most of the methods cited are regarded as part of the statistical methodology of multivariate analysis, concerning data with many variables. Multidimensional analysis is a useful term for describing methods where the results of analysis are multidimensional whereas the data themselves are not necessarily multivariate - e.g. correspondence analysis, where there are only two (categorical) variables.

horizontal and vertical axes (the shape parameter of a graph, see Cleveland and McGill, 1987). Sometimes, it is clear that interpretations are unjustified. Sometimes, it is not clear precisely what method of analysis has been used, leading to the impossibility of deciding whether or not interpretations are valid. All of these problems severely undermine the important advantages of graphical representations: rapid interpretation and communication of complex information.

There are many types of perceptual map, each with their own considerations that should be taken into account when constructed. The aim of this paper is to give guidelines for the presentation of perceptual maps so that their interpretation is best facilitated. We also propose the use of icons to provide interpretation guidance. Potentially these icons allow readers to correctly and confidently interpret a map even if they are unfamiliar with the statistical technique used to create the map. Nowadays perceptual mapping is often used in marketing practice and less frequently in marketing research (Huber, 2008), so we shall primarily concern ourselves with current practice in the applied marketing literature. Moreover, we will not concern ourselves with the question whether or not an inappropriate analysis has been used; rather, our aim is to ensure that whatever analysis underpins the perceptual map, the map is presented with clarity.

To achieve our aim, we shall first stipulate which properties of maps are necessary and/or desirable. There exist several important and useful references that describe general principles of graphs (e.g. Tufte, 1983; Wainer, 2005; Cleveland and McGill, 1987). However, these treatises do not specifically address issues specific to the multidimensional methodology yielding the perceptual maps frequently encountered in the marketing literature. We will fill this gap by providing some general principles or desiderata for such maps in Section 2. Then, using these desiderata we explore, in Section 3, the extent to which these ideals appear to be met in applied marketing journals. As already mentioned, however, our findings suggest that there is room and need for improvements. The diversity in criteria and method can lead to significant differences in the interpretation of perceptual maps. For example, sometimes distances between certain points have a clear interpretation whereas in other cases, distances cannot be interpreted directly. To facilitate an immediate interpretation of different perceptual maps, we introduce an additional aid to perceptual map interpretation in Section 4; the use of iconography to indicate permissible interpretation strategies. Finally, in Section 5 we give our conclusions. We also give, in an appendix, a brief summary of various multidimensional techniques along with an indication of appropriate interpretation strategies that might be applied to the perceptual maps they produce.

2 Desiderata for perceptual maps

The construction of any perceptual map requires many graphical design decisions to be taken, either by the creator of the map or automatically by the generating software. For example, the style and scale to be used on axes; the labelling to be applied to points and/or lines; and the text of any title or captions. Though trivial sounding, such decisions are important as they impact on the ability of the map to clearly and accurately represent the underlying data. Desiderata for a well-designed perceptual map are summarised in Box 1 below.

Box 1 Desiderata for perceptual maps

1. Include a caption or title.
2. Include a legend or key when there are two or more types of points or lines.
3. Ensure that the shape parameter is 1.
4. Indicate the origin when it is required for interpretational purposes.
5. Label points.
6. Avoid clutter.

That these are desiderata of good perceptual maps may seem obvious. However, even with simple plots such as bar charts and scatterplots, examples of poor graphical design, and the consequent misleading impact on the message that the plot conveys, have been described extensively elsewhere, for example, Wainer (2005) and Tufte (1983). They contain numerous examples that have appeared in newspapers and elsewhere which, whether by accident or design, give a misleading impression. With such examples they also show how revised well-designed graphics can enhance the message in the underlying data. The impact of graphical design can be at least as important with perceptual maps as failure to follow these desiderata can render a map useless.

In Section 2.1, we will discuss in more detail Desiderata 1 and 2. In particular we will show that it is not always possible to deduce the exact type that a perceptual map is just by looking at it. Desiderata 3 and 4 are discussed further in Section 2.2. Our final two desiderata are discussed in more detail in Section 2.3.

We will illustrate the impact of good graphical design on perceptual maps with reference to a particular data set about patterns of consumer satisfaction in the EU. In 2007 a report was published on the satisfaction of consumers in the EU with a range of eleven services, from

Table 1: Percentage satisfied consumers with eleven services for citizens in the European Union

Country	Service										
	Air	Mob	Ins	Bank	Water	Gas	Elec	Post	Phone	Coach	Bus
Austria (AT)	82.2	76.4	76.1	77.3	86.8	75.9	79.5	57.9	70.1	49.0	61.0
Belgium (BE)	59.7	74.4	74.3	75.9	64.3	65.2	65.2	43.8	62.0	52.2	57.1
Czech Republic (CZ)	75.7	73.8	65.1	67.4	59.2	64.5	58.3	51.8	40.7	43.7	42.7
Denmark (DK)	65.0	71.4	73.6	71.7	84.8	77.9	78.9	52.8	61.3	37.5	42.1
Estonia (EST)	64.1	78.3	70.7	84.2	51.5	68.1	71.8	71.5	71.0	58.0	49.7
Germany (D)	83.2	83.5	81.4	78.6	83.9	69.7	72.7	58.5	71.4	42.4	52.3
Greece (GR)	76.2	70.9	68.4	65.3	59.0	87.4	48.1	69.7	45.1	63.4	53.6
Finland (FI)	77.5	72.7	75.2	87.4	79.6	76.6	63.2	66.4	60.9	67.4	63.9
France (FR)	56.1	55.3	65.3	55.5	52.0	58.2	60.4	46.1	49.3	52.6	49.2
Hungary (HU)	82.5	82.9	71.3	74.4	70.6	64.9	72.6	70.7	65.2	50.0	37.7
Ireland (IRL)	67.0	73.0	81.3	67.8	71.5	81.8	73.2	81.7	78.3	72.4	65.6
Italy (I)	51.1	49.7	42.3	36.7	40.4	36.4	34.8	28.7	27.4	24.2	33.7
Latvia (LV)	68.0	79.9	70.6	80.6	50.5	67.5	73.1	67.8	65.1	66.2	62.0
Lithuania (LT)	78.6	79.1	75.7	80.3	59.6	84.5	81.6	80.2	72.4	66.5	57.4
Luxembourg (LU)	66.5	69.2	74.7	75.2	73.5	69.3	71.5	67.5	68.7	54.2	57.5
Netherlands (NL)	52.9	51.0	46.6	50.5	53.4	47.6	41.1	47.5	39.7	24.0	25.0
Poland (PL)	67.7	67.2	55.7	62.0	56.8	58.9	59.9	61.3	45.2	46.2	39.2
Portugal (PT)	64.1	68.8	50.3	64.7	52.2	49.5	36.4	64.9	29.7	55.5	40.4
Slovakia (SK)	62.0	73.0	58.1	64.7	52.8	43.3	52.8	59.4	53.1	28.6	21.9
Slovenia (SVN)	73.8	76.8	73.9	75.2	72.0	77.2	73.5	74.2	68.5	51.5	46.4
Spain (ES)	45.0	41.7	54.7	50.9	46.9	47.7	42.7	47.7	36.0	43.6	35.4
Sweden (SWE)	62.4	71.7	67.1	80.3	80.7	74.1	53.2	44.8	63.4	54.1	39.5
United Kingdom (UK)	67.6	67.5	67.6	67.2	59.7	58.3	58.2	55.8	59.8	55.4	43.3

Air — Air transport, Mob — Mobile phone, Ins — Insurance, Bank — Retail banking,
 Water — Water, Gas — Gas, Elec — Electricity, Post — Postal services,
 Phone — Fixed Phone, Coach — Extra urban transport, Bus — Bus

electricity and urban transport to retail banking and insurance (IPSOS-INRA, 2007). Part of these data were then summarised by the percentage satisfied on a nation by nation basis for 23 EU countries. For reference these data are given in Table 1. A question therefore arises as to the nature of differences in consumer satisfaction across services and/or between nationalities.

2.1 Its obvious what sort of map it is ... isn't it?

Just because you know what a plot represents when you create it, does not mean that everyone else will when they come to look at it. This is particularly true of perceptual maps generated by multidimensional techniques. Different types of maps, with different interpretations, can look superficially the same. For example two perceptual maps of the EU consumer satisfaction data are given in Figure 1.

Superficially both perceptual maps look similar. In both, 23 points, each representing a country are plotted with respect to a pair of axes. However, the configuration of the points in the maps are different. For example, the point representing Italy is to the extreme right hand side of one map, but appears centrally in the other map.

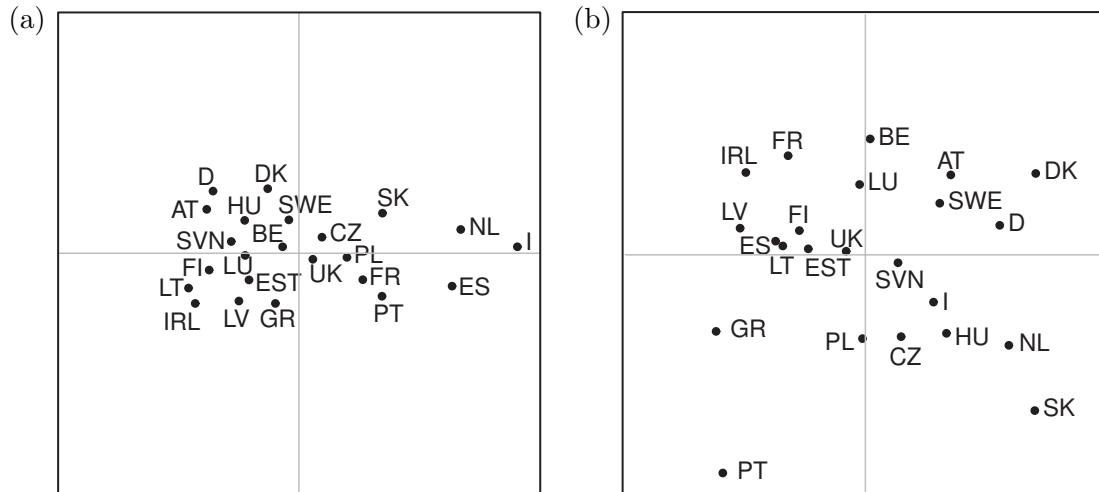


Figure 1: Two perceptual maps

The discrepancy between the two maps in Figure 1 arises because the two perceptual maps were generated using different multidimensional techniques, that are approximating different quantities, and hence have different interpretations. Without an adequate caption neither diagram is interpretable. Readers are left unsure as to the type of map, and hence are invited to guess what interpretations are valid. This ambiguity is avoided if Desideratum 1 in Box 1 is followed in an appropriate way.

So what should go in a caption? As general rule the caption should convey enough information to allow a reader who is in possession of the data (and suitable software) to recreate the perceptual map. Obviously this includes the multidimensional technique used to generate the map (e.g. PCA, MDS, etc). It also includes which variant of the technique was used (e.g. the type of non-metric MDS). Any preprocessing applied to the data (e.g. standardisation and/or distance function used) should also be mentioned in the caption. In some circumstances it may be helpful to mention the algorithm used to implement the multidimensional technique, such as ALSCAL or MDPREF. However, this should not be seen as a substitute for specifying the technique the algorithm is performing. Thus an appropriate caption for Figure 1 is as follows.

Figure 1: (a) A plot of the scores for the first two principal components, based on an analysis of standardised data. (b) A plot of the row points from a correspondence analysis using row principal or principal normalisation ($\alpha = 1$ in Appendix A.3).

Similar issues also arise when a perceptual map contains plotted symbols of two or more different types. For example, Figure 2 is a perceptual map of the consumer satisfaction data on which both countries and services are depicted. From the map itself it is clear that two types of things are being plotted, one thing with points and the other with lines. In this case the

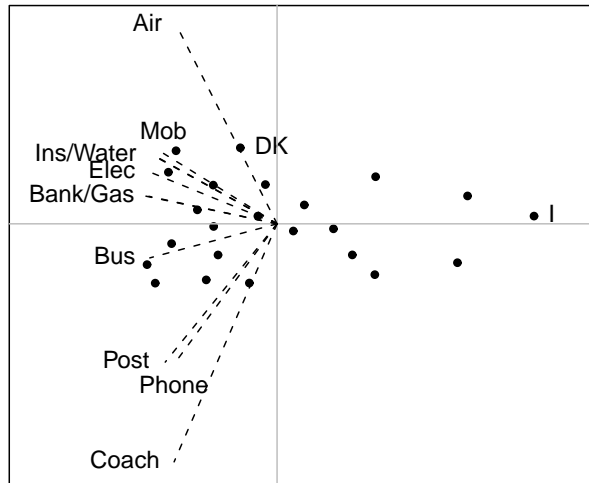


Figure 2: An interpolative biplot, based on an principal component analysis of standardised data. The length of each line representing a service corresponds to 10 units after standardisation.

caption happens to give a further clue, namely that the lines represent services. However, the reader is still left to guess what the points represents. And although many would recognise the abbreviations for countries, there is no guarantee that all readers of the plot would. Hence Desideratum 2 in Box 1. A legend added to Figure 2 would to make it unambiguous that points represent countries and (dotted) lines represent services. Alternatively, the information can be included in the caption. For example, Figure 2's caption could be amended in the following way.

Figure 2: An interpolative biplot, based on an principal component analysis of standardised data. The length of each line representing a service corresponds to 10 units after standardisation. '●' represents countries and '- - -' represents services.

2.2 The bigger, the better?

To make a perceptual map requires ' x ' and ' y ' coordinates for individual plotted elements such as points or lines. When the interpretation of a perceptual map only requires the relative, not absolute, positions of the plotted points, lines and the angles they subtend at some origin, the choice of the x and y coordinates used, is arbitrary with respect to rotation. When the x and y coordinates are reified (that is they are interpreted as meaningful latent variables) then rotation is admissible when governed by factor-analytical considerations such as an appeal to the concept of simple structure. Factor rotation is beyond the scope of the present paper.

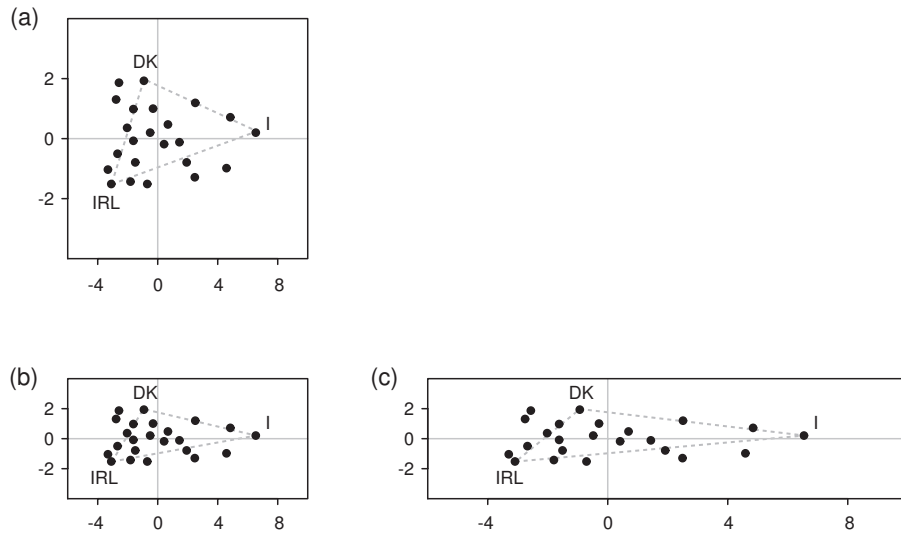


Figure 3: Three variants of Figure 1(a) (PCA) constructed so that only the shape parameter differs: (a) has shape parameter 2, (b) has shape parameter 1 and (c) has shape parameter 0.5. The grey dotted lines are just auxiliary lines highlighting the distances between three points.

These x and y coordinates are then translated to positions on a page (or screen) via x and y axes. The scaling used for the x and y axes defines the shape parameter of the map: the ratio of the length of one unit along the y axis to the length of one unit along the x axis. The shape parameter affects the way in which interrelationships between points (and axes) are perceived. In particular it changes the perceived angles and distances between objects in the map. Only with a shape parameter of one do the angles and distances in the map match those that can be mathematically calculated from the data, hence Desideratum 3 in Box 1. Furthermore, the interpretation of some perceptual maps relies on the angle points and/or lines subtend with the origin. Obviously then, on such maps it is important that the position of the origin is indicated, hence Desideratum 4 in Box 1.

For example, Figure 3 gives three different variants of the same perceptual map. Figure 3(b) is constructed so that one unit along the x axis is the same length as one unit along the y axis. Thus the map's shape parameter is exactly one. Notice in this map three countries which are extreme with respect to their consumers' satisfaction with services: Italy (I), Denmark (DK) and Ireland (IRL) though using just this map it is not possible to suggest exactly what it is about these countries' consumer satisfaction scores that makes them unusual. Furthermore, the point representing I is more distant from the point representing IRL than the point representing DK. Thus the plot suggests that there are greater differences in satisfaction with services between Italian and Irish consumers than there are between Danish and Irish consumers.

In Figure 3(a) a shape parameter of 2 is used. Note now that the data appear to be spread

equally in all directions, with the points labelled I, DK and IRL at the vertices of an approximate equilateral triangle. Although this might look more aesthetically pleasing, it has seriously distorted the true distances, for example making DK and IRL look relatively more distant than they are. In contrast in Figure 3(c), with its shape parameter of 0.5, the distortion is different, for example by making the point representing Italy look even more extreme than it is.

The distortion of distances and angles induced by the shape parameter is hard to overcome. Labelled scales can indicate the degree of distortion as well as merely its presence. For example, the labelling of the scales in all the maps of Figure 3 indicate accurately the shape parameter used in each map. But labelling, however good, cannot overcome the immediate visual impression. And, after all, creating visual impressions of data is the whole point of constructing such maps in the first place. For this reason, as a general rule, a shape parameter of one should always be used for perceptual maps.

2.3 And the point is?

In its most basic form, a perceptual map consists of just a set of points and/or lines on a two-dimensional plot. As discussed in Section 2.1, a caption or title is required so that it is clear to the reader how the map should be interpreted. However the caption or title usually only gives general guidance about how that particular type of perceptual map should be interpreted; beyond that it does not help with reading the specific message contained within the map. Hence Desideratum 5 in Box 1: label points.

For example Figure 4(a) shows the positions of the row points for the consumer satisfaction data from a correspondence analysis using row principal normalisation or principal normalisation ($\alpha = 1$ in Appendix A.3). But without further labelling it is impossible to see little more than there is variation in the profiles of consumer satisfaction for the different countries. We do not even know that the points represent countries.

The use of codes for the different countries as in Figures 4(b) and Figure 4(c) makes it possible to compare countries. It is now clear that Portugal (labelled 18/PT) is the country that is most different to the rest as its point lies apart from the other points in the lower left hand corner of the plot. Of these, the labelling in Figure 4(c) is to be preferred to that used in Figure 4(b), because it provides a more intuitive link between the labels and the countries they represent. Furthermore, the use of labels in addition to the plotted points, rather than instead of, reduces ambiguity in the positioning of each data point even if the labels used are long.

However, though labelling is important, too much labelling can detract from the message contained in the pattern of the data. Thus the use of labelling has to be balanced with our final desideratum for perceptual maps, Desideratum 6: avoid clutter. One way of doing this, is to

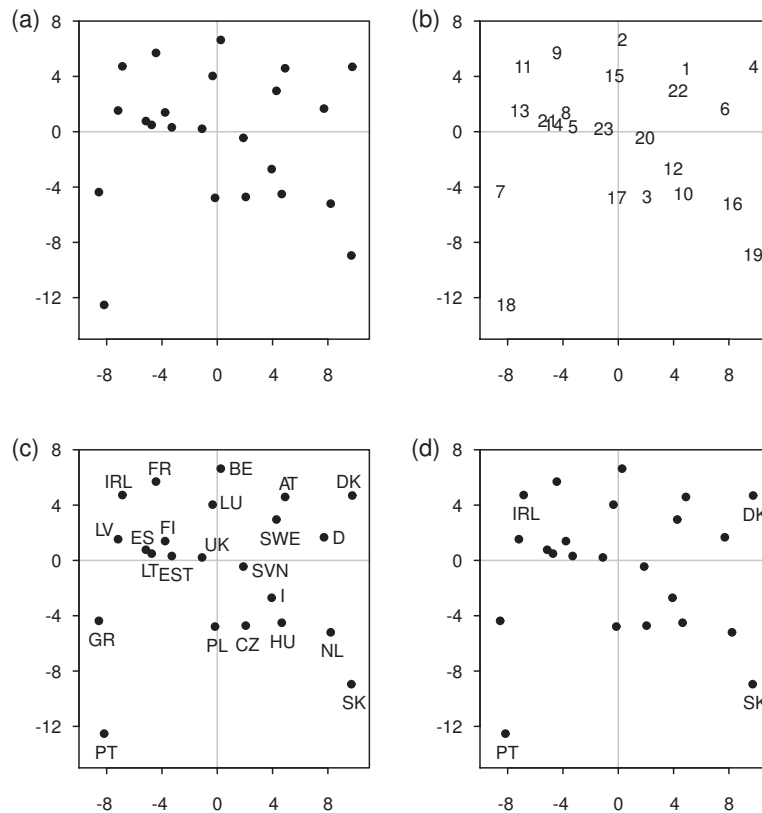


Figure 4: Four perceptual maps which differ only in the degree of labelling. All four maps were constructed using correspondence analysis with a row principal or principal normalisation ($\alpha = 1$ in Appendix A.3).

label only some of the data points as in done in Figure 4(d). Notice that the pattern of the points stands out more than it does in Figure 4, whilst the identity of a few key points is not lost. The choice of which labels to keep is necessarily a subjective one, driven by which data points are particularly noteworthy. For example, in Figure 4(d) just four points are specifically labelled, selected because they are outlying. Colour can also be used effectively to reduce clutter on a map. Notice that on all the plots, Figures 4(a) to 4(d), the lines representing zero values on the x and y axes are included, but using grey, not black. In this way, the position of the origin can be conveyed (Desideratum 4) whilst allowing the data points to dominate the visual impression. Apart from confirming the unit shape parameter, the x and y scales might even be regarded as clutter and removed without loss.

3 Current situation

In the previous section we highlighted six desiderata for presenting perceptual maps. A question therefore arises as to what extent these desiderata are respected in practice.

We conducted a survey of recent articles in marketing journals with the aim of seeing how often published perceptual maps currently match our desiderata. For the survey, a Google Scholar (<http://www.scholar.google.com>) search was conducted to identify articles published between 2005 and 2008 that contained perceptual maps. The search term was “perceptual map” and we constrained our results to results from the (Google Scholar) subject areas “Business, Administration, Finance, and Economics”. This yielded a total of approximately 180 results of which about half were eliminated from further consideration because they did not clearly relate to papers in academic journals. After eliminating those papers that contained no plots, 59 papers remained. Of these, 1 was double counted, 1 finally came out in 2009, and 5 did not contain perceptual maps after all. This left a total of 52 papers and 114 perceptual maps that were examined in detail. These 114 plots can be split up according to the used methodology. We categorised the methods as follows: CA, correspondence analysis, 27 (24%) plots in total. MCA, multiple correspondence analysis, 8 (7%) plots. PCA, principal component analysis (including categorical PCA and factor analysis), 13 (11%) plots. MDS, multidimensional scaling, 28 (25%) plots. Miscellaneous, maps based on discriminant analysis, nonlinear canonical correlation analysis, neural networks and some undetermined but in the paper referenced methodology, 17 (15%) plots. Finally, we include a category “Unknown” for those perceptual maps when we could not establish which method had been used; there were 21 (18%) of such plots. Note that our categorisation of the methods used is based on what the authors claimed they were doing. Unfortunately this means that there is no guarantee that the methodology the authors actually used fits in with the brief descriptions we provide in the appendix.

Table 2: Relative frequencies of observed flaws with respect to desiderata, classified by method

Method	Prop. maps	Desideratum					
		1 Caption	2 Legend	3 Shape	4 Origin	5 Label	6 Clutter
CA	0.24	0.44	0.59	0.22	0.04	0.00	0.11
MCA	0.07	0.75	0.38	0.75	0.00	0.00	0.38
MDS	0.25	0.57	0.57	0.68	0.04	0.00	0.50
PCA	0.11	0.85	0.39	0.54	0.39	0.31	0.08
Misc	0.15	0.65	0.24	0.41	0.00	0.12	0.35
Unknown	0.18	1.00	0.43	0.43	0.10	0.10	0.24
Overall	1.00	0.68	0.47	0.47	0.08	0.07	0.28

Prop. map — Proportion of perceptual maps, Misc — Miscellaneous,
Caption — Inadequate or missing caption,
Legend — inadequate or missing legend,
Shape — Shape parameter $\geq 5/4$ or $\leq 4/5$,
Origin — origin not indicated, Clutter — appears cluttered

In our appraisal, we restricted ourselves to observable flaws with regards to the desiderata given in Box 1. Obviously, some degree of subjectivity can never be avoided when recording the flaws. In particular, deciding whether the legend and labels are sufficient and if the chart is cluttered, always depends to some degree on the reader’s subjective judgement. We therefore recorded them as conservatively as possible. That is, in case of doubt, we did not record a problem. The results are summarised in Table 2 where the relative frequencies of occurrences are classified by method.

- Include caption or title (Desideratum 1): We recorded whether the method, the variant, or the algorithm was mentioned in the caption. Only if none were mentioned, did we consider the caption flawed. Nonetheless, using this conservative coding, we still found that 68% of the published perceptual maps contained captions without mention of method, variant or algorithm. Moreover, for 21 perceptual maps (18%) the methodology used could not be established at all. That is, even in the text no mention other than the generic term perceptual map, was provided. These 21 plots are labelled unknown. Although there are differences between the relative frequencies, it appears that the problem of insufficient captioning is quite severe for all methods. The relative frequencies are smallest for CA and MDS. Still, nearly half of the CA plots did not contain proper captioning. This is a large number especially if you take into account that there are many variants of CA, each leading to different maps requiring different interpretations (see Appendix A.3).
- Include legend or key (Desideratum 2): We considered the legend to be flawed when it was either absent or providing insufficient information to understand the map. In Table 2 we see that for many maps the legend does not provide what is needed. Overall, nearly 50% of the legends were insufficient.

- Ensure shape parameter is one (Desideratum 3): As explained in Section 2.2, the shape parameter for a perceptual map should always equal one. To see whether this is the case in practice we calculated, where possible, the shape parameters by dividing the y scale by the x scale. The resulting shape parameters were considered to be unequal to one when the calculated parameter was either smaller than, or equal to, $4/5$ and, similarly, larger than, or equal to $5/4$. As obvious as it may seem to have a shape parameter of one, and given that this property is often emphasised in the literature at numerous occasions (Gower and Hand, 1996; Greenacre, 2007; Borg and Groenen, 2005), our sample indicates that in nearly half of the published plots, the shape parameter is not one. In fact, this is a conservative estimate as for 22% of the maps we couldn't establish what the shape parameter was. In a perfect world, not being able to verify the shape parameter would not be a problem as all perceptual maps would have a shape parameter of one. Unfortunately, however, we do not live in a perfect world. Yet.
- Indicate the origin (Desideratum 4): We conservatively recorded wrongful omissions of the origin. That is, only in cases where the origin was clearly essential but not present, did we record this as a flaw. The results are collected in Table 2. At first sight, omission of the origin does not appear to be a big problem with wrongful omissions in only 8% of the plots. However, it should be noted that this type of problem is only applicable for certain methods. In PCA, angles typically do play an important role and an origin is useful for representing an average sample, whereas in MDS the origin is typically irrelevant. Omission of the origin in 30% of the PCA perceptual maps indicates that current practice is far from satisfactory.
- Label points (Desideratum 5): Labelling appears to be less problematic. In most plots point were properly labelled. In fact, the (multiple) correspondence analysis and multi-dimensional scaling plots all contained appropriately labelled points. Sometimes software overprints labels of adjacent points, it is useful to have the interactive capability to rear-range labels.
- Avoid clutter (Desideratum 6): The occurrence of chart clutter was recorded by visual inspection. In 28% of the maps we found the amount of clutter to be problematic.

To call these numbers disappointing would be an understatement. Consider for example the multidimensional scaling maps. Assuming that the plots with unknown shape parameters are in fact correct, 71% of the maps are distorted, prohibiting, or, at its best, severely complicating a meaningful interpretation. Recall that the maps analysed here are all taken from scientific journals. This means that in addition to the journal's editor, the publications have been reviewed by usually two or more field experts before publication. The severity of this problem leads one to wonder about the reasons. One possible explanation could be related to the software used to prepare the maps. Several papers we reviewed used standard software leaving

Table 3: Shape parameters of Perceptual Maps obtained using standard software packages. A ‘-’ indicates that the shape parameter is not always one, ‘+’ indicates that we did not encounter non unit shape parameters, and n.a. stands for ‘not available’ in the package.

Software	Version	PCA	MDS	CA	MCA
SPSS	17	-	+/- ¹	+	+
MiniTab	15	-	n.a.	+	+
XLStat	2010	-	-	-	-
Stata	10	+/- ²	+	+	+
GenStat	12	+	+	n.a.	n.a.

¹ SPSS: Alscal = -, Proxscal = +

² Stata: PCA = -, Biplot = +

their authors with little or no control over the properties of the graphical output. Moreover, in some cases, the output of such programs appears to be flawed, in particular with respect to the shape parameter. Understandably, software users tend to use default options. This is especially true of those who are inexperienced in statistical methodology and do not have the background to recognise when the default is appropriate, let alone how to override it, even when the capability is provided. Although a thorough review of available software is beyond the scope of this paper, we inspected the output from a selection of popular statistical software programs for four popular perceptual mapping methods; PCA, MDS, CA and MCA. The results are summarised in Table 3.

Many deficiencies in output can be ameliorated by providing additional information in the caption to a figure. However, correcting an incorrect shape parameter is more of a problem. With some software control is impossible, with others it may be controlled but with various degrees of difficulty. Nevertheless users should make themselves aware of the possibilities for controlling the shape parameter and/or switch to software that produces maps with a shape parameter of one. Another hazard is that a figure with the correct shape parameter may be adjusted in the printing production process so as to fit elegantly into the printed page. Editors should be aware of the problem and, when necessary, insist on rectification.

4 Iconography to aid interpretation

The desiderata described in Section 2 ensure that readers of perceptual maps know what is being displayed (via appropriate captions, legends and labels) and that the display is not distorted (because the shape parameter is one). However, these desiderata are, by themselves, insufficient to ensure that a reader can correctly interpret the map. Knowledge about how the plot can *and cannot* be interpreted is also required by the reader. For example, the reader needs to know whether it is valid to make statements based on the distance between two points










in the map, something which depends on the multidimensional technique used to create the map. Currently there is the implicit expectation that this knowledge arises because readers are sufficiently conversant with the multidimensional technique used to produce the map. However, in the face of the variety of multidimensional techniques which underlie perceptual maps, each typically accompanied by a range of options, and implemented using a range of different software and algorithms, such an expectation is unrealistic. There is thus a clear need to include interpretational guidance with every perceptual map.

We feel that creators of perceptual maps are the ones who are best placed to give this interpretational guidance. We propose that this is done using iconography, using icons to denote the means by which the map can be interpreted. If widely adopted and recognised, these icons are a vehicle for directly passing interpretational guidance directly from map creator to reader. The reader then does not have to know the intricacies of the multidimensional technique used by the map creator and can concentrate on the message the map is conveying.

Two key means of interpreting perceptual maps are distance and angle. Put quite simply, distance-based interpretations are those in which the plotted distance between points translates directly to the implied similarity between them. Maps derived from different techniques can also vary in whether distances or angles can only be compared between points of the same set (for example, respondents) or between sets (for example, between respondents and product points). Table 4 presents the icons and their interpretation that we propose. Each icon either depicts admissible or non-admissible interpretations in a particular perceptual map. Two solid circles indicate that the relation concerns points of the same set, whereas an open circle combined with a solid one indicates that the relation holds between points of different sets. Negation of these icons is also possible. Such icons provide a proactive means of warning readers when such interpretations are tempting but should not be made. We also include an icon to indicate when the shape parameter is one. This can be used to reassure readers that shape parameter of the perceptual map is appropriate, even in situations where the shape parameter cannot be verified.

We illustrate the utility of the proposed iconography via two examples. In the first example, given in Section 4.1, correspondence analysis (CA) will be used to create some perceptual maps. CA is particularly suited for illustrating the usefulness of iconography as the interpretation of a CA plot depends on the chosen variant of CA. In particular, different CA normalisation schemes require different interpretations. Hence, unless the variant is clearly stated *and* the reader is familiar with the interpretational implications of that variant, interpretation is compromised. But just by adding icons, it becomes evident how and what can be interpreted in the CA plot. Our second example, given in Section 4.2, concerns a perceptual map constructed using unfamiliar methodology. This means that readers are likely to be unsure of the appropriate interpretational rules that apply to the perceptual map. Our iconography, however, allows all

Table 4: Icons that indicate how the relations between points, vectors or lines can be interpreted appropriately.

Icon	Interpretation
	The plot has shape parameter 1.
	Distances are interpretable between two points of the same set.
	Distances are not interpretable between two points of the same set.
	Distances are interpretable between two points of the different sets.
	Distances are not interpretable between two points of the different sets.
	Projections are interpretable between two vectors of the same set.
	Projections are not interpretable between two vectors of the same set.
	Projections are interpretable between two vectors of the different sets.
	Projections are not interpretable between two vectors of the different sets.

to immediately interpret the perceptual map.

4.1 Example: Correspondence Analysis

Correspondence analysis (CA) is a popular perceptual mapping method that, at least mathematically, can be applied to any nonnegative data matrix. It is, however, particularly suited for investigating the deviations from independence in a two-way contingency table. As an example, we consider the analysis of Table 5, a contingency table of food store usage by 700 consumers reproduced from Greenacre (2007).

Table 5: Contingency table of food store usage by age group reproduced from Greenacre (2007).

Food store	Age group				Total
	16-24	25-34	35-49	50+	
A	37	39	45	64	185
B	13	23	33	38	107
C	33	69	67	56	225
D	16	31	34	22	103
E	8	16	21	35	80
Total	107	178	200	215	700

In CA row and column categories are represented by points in a low-dimensional space. The points are positioned so that as much as possible of the deviation from independence, the so-called inertia, is captured. For some technical details on CA we refer to the appendix. One difficulty with CA is that similar looking maps, which have different interpretations, are possible. The basis for many of these differences lies in which normalisation is used. That is, how exactly the inertia per dimension (or eigenvalue) is spread over the row and column points. Normalisations that are used include the following.

- Column principal normalisation ($\alpha = 0, \beta = 1$ in Appendix A.3). The inertia is spread over the column points.
- Symmetric normalisation ($\alpha = \frac{1}{2}, \beta = \frac{1}{2}$ in Appendix A.3). The inertia is equally distributed between row and column points.
- Principal normalisation ($\alpha = 1, \beta = 1$ in Appendix A.3). Here the inertia is spread twice, once over the row coordinates and a second time over the column coordinates. This type of map is quite popular in the French literature of correspondence analysis.

With column principal normalisation, the squared Euclidean distances between the column points can be interpreted as chi-squared distances. For example in Figure 5(a), looking only at the age groups (the open circles), it can be seen that the 25–34 and 35–49 age groups have similar patterns of food store usage because the two points representing these age groups are relatively close together. The more distant positions of the other two age group points suggests that the two other age groups seem to be equally different from these two middle age groups as they are from each other. The same interpretation of distances between column points is valid when principal normalisation is used, but it is *not* valid when symmetric normalisation is used.

The interpretability, or otherwise, of distances between age group points can be indicated by the presence or absence of the icon $\circ\text{---}\circ$ in the legend. Thus in the legends of Figures 5(a) and 5(c) (the plots created using column principal and principal normalisations) the icon $\circ\text{---}\circ$ appears, but it is missing from the legend of Figure 5(b).

The appearance of the icon $\bullet\text{---}\bullet$ only in the legend of Figure 5(c) indicates that a second type of distance interpretation is possible in this plot, but not the other two: the interpretation of distances between food store points as chi-squared distances. So, for example, from Figure 5(c) is it valid to say that the age profiles of users of food stores C and D are the most similar because the points representing these two stores are closest together on Figure 5(c). Such an assertion cannot be validly made from Figures 5(a) and 5(b).

CA can also be thought of as a decomposition method that makes use of inner products. Thus some CA plots can be interpreted by projecting row points on lines through column points (or

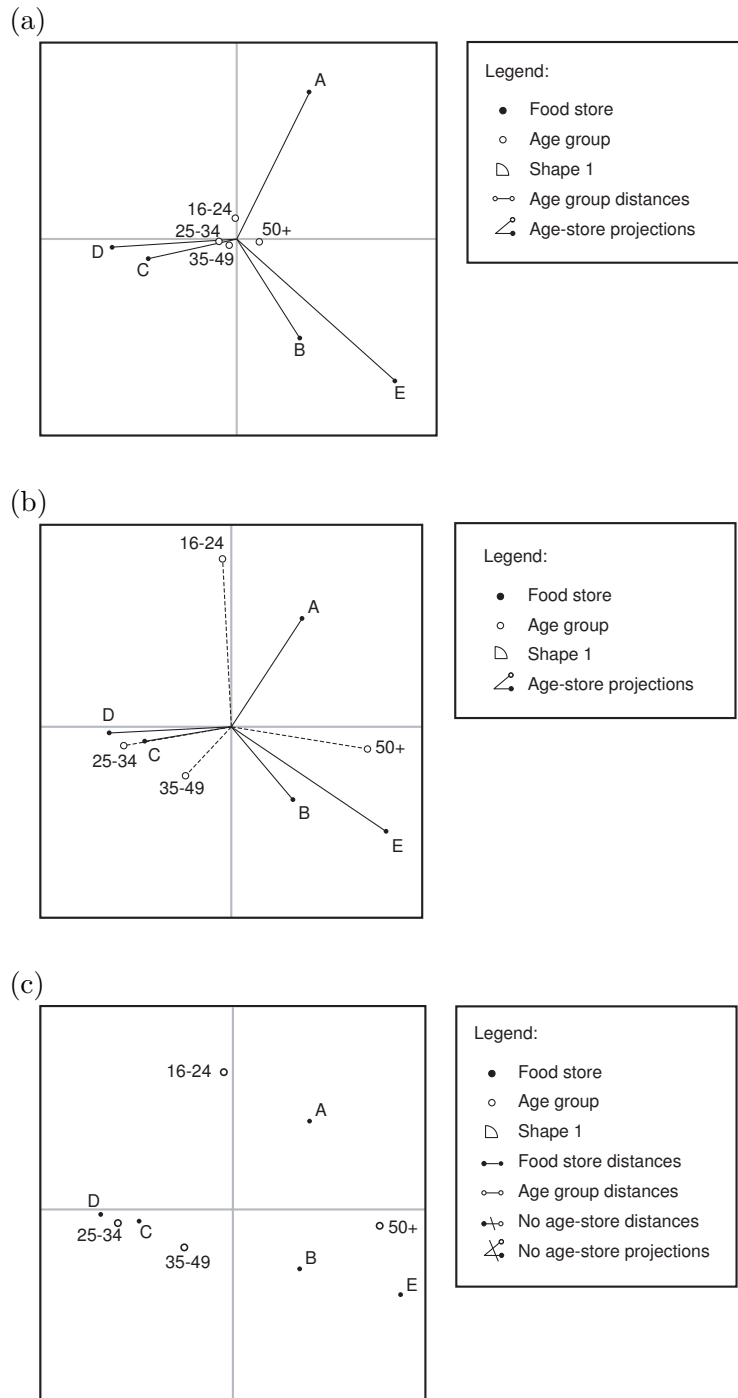


Figure 5: Correspondence analysis solution both row and columns in (a) column principal normalisation ($\alpha = 0, \beta = 1$ in Appendix A.3), (b) symmetric normalisation ($\alpha = \frac{1}{2}, \beta = \frac{1}{2}$ in Appendix A.3) and (c) principal normalisation ($\alpha = 1, \beta = 1$ in Appendix A.3). The data are those from Table 5, a contingency table of five foodstores (rows) and four age groups (columns).

vice versa). In the legend, the presence of the icon \curvearrowright can be used to indicate when projections between row and column points are interpretable. Thus, comparing legends, it is clear that such projections are valid for Figures 5(a) and 5(b), but not Figure 5(c). For example, suppose we are interested in the age profile of shop A users. We can investigate this by considering the projection of each of the age groups points onto the line through the origin and point A in Figure 5(b). The highest projection along this line occurs with the point representing the 16–24 age group indicating that there are more people than expected visiting shop A, than for any other age group. Note that the actual proportion is smallest in the 16-24 age group. The point representing the 50+ age group also projects on the positive side of this line, so that shop A receives more than expected people from the 50+ age group. The 25–34 and 35–49 age groups visit shop A less than expected as the projections of points representing these two age groups fall on the negative side of the line through the origin and A. Figure 5(a) leads to the same interpretation, but the scaling makes the process more difficult.

Another thing to notice about Figure 5 is that we have dispensed with the calibration and labelling of the horizontal and vertical axes. This is because now that we have the icon \sqsupset to inform us that the shape parameter is one, calibration is superfluous. One would retain the labelling, and perhaps calibration, only when discussing possible reifications of these directions.

Finally, note that the negated icons \curvearrowleft and \curvearrowright are given in the legend of Figure 5(c). This is done to stress the uninterpretability of the distances between points age group and food store points and the uninterpretability of the projection of age group points on the lines to the food store points (and vice versa). In other words, on this plot it is not valid to directly compare the age group and food store point configurations, regardless of how tempting it is to do so. (However, Figure 5(b) and Figure 5(c) appear very similar. Thus invalid projection interpretations of Figure 5(c) would not be overly misleading in this case.)

4.2 Example: Perceptual Maps for Unfamiliar Models

The proposed iconography is particularly useful to aid interpretation of perceptual maps arising from unfamiliar models. As an example, we take the clusterwise bilinear MDS model used by DeSarbo et al. (2008). This model takes data in the form of preferences, considerations, or intentions of consumer i to buy brand j and produces a perceptual map showing the clusters and the brands. It can be seen as an adaptation of k -means clustering with a low rank restriction on the matrix of means per cluster (segment) and brand.

DeSarbo et al. (2008) apply the model to data obtained from a US automotive consumer research study that asked 360 consumers their buying intentions for 18 compact SUVs scored on a four point scale ranging from 1 (definitely would not consider) to 4 (definitely would consider). In

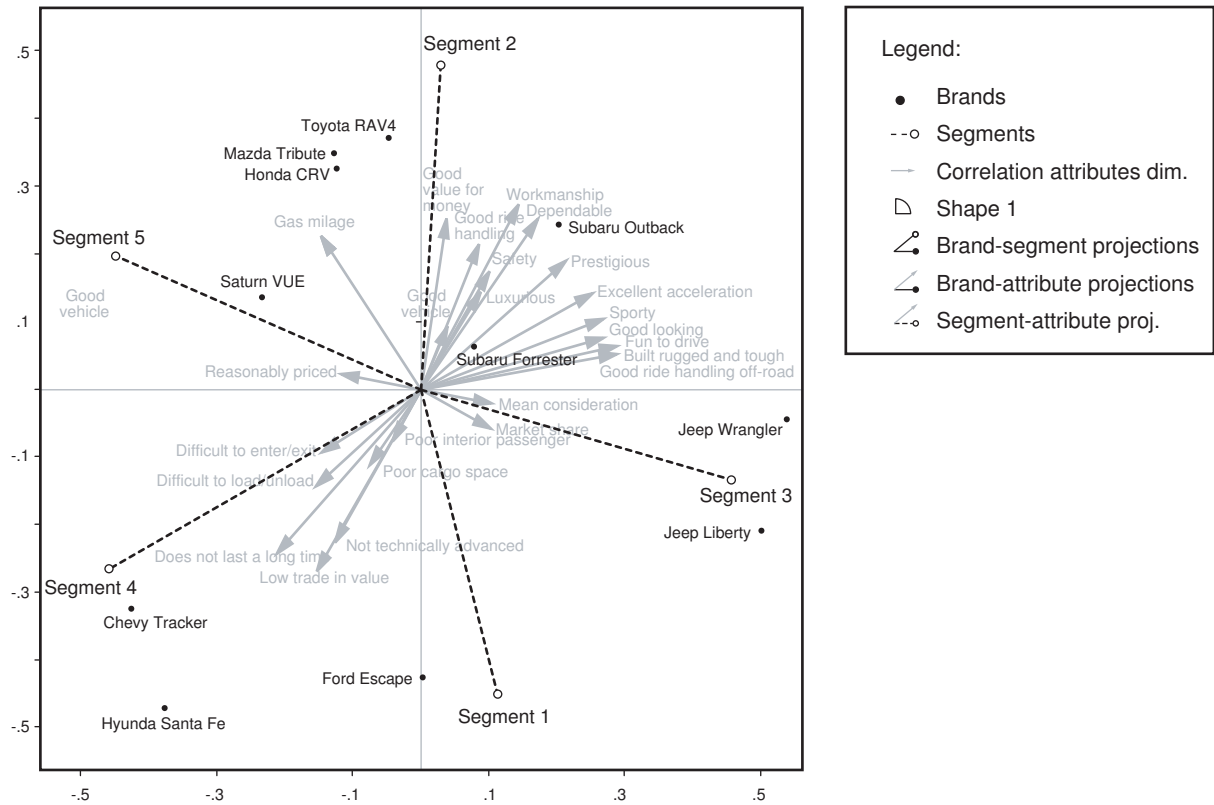


Figure 6: Perceptual map of the clusterwise bilinear MDS model proposed by DeSarbo et al. (2008) on SUV preference data. This figure shows dimensions 1 and 2 and is adapted from Figure 2 of DeSarbo et al. (2008).

in addition, data about 25 attributes were gathered for each SUV. The resulting map is shown in Figure 6.

Using our iconography, the reader does not need to know the details of DeSarbo’s clusterwise bilinear MDS model in order to appropriately interpret Figure 6; admissible interpretations are indicated by the icons of the form \curvearrowright . For example, the projections of the two Jeeps and the Ford Escape are high and positive on the line of Segment 3. So Segment 3 is characterised mainly by the two Jeeps and somewhat the Ford Escape. Similarly Segment 1 is characterised by the Ford Escape and is unlike the Toyota RAV4, Mazda Tribute, and Honda CR-V.

Figure 6 also shows the (scaled) correlations of 25 attributes with the dimensions (in marketing called property fitting). The icons show that these attributes can be interpreted in two ways; by the projection of brands on attributes and by the projection of segments on attributes. For example, ‘Gas mileage’ correlates positively with the Toyota RAV4, Mazda Tribute, and Honda CR-V, but negatively with the two Jeeps and the Ford Escape. Similarly ‘Gas mileage’ correlates positively with Segments 2 and 5 and negatively with Segments 1 and 3.

5 Conclusion

We have been critical about the quality of graphical presentation in the marketing literature but it should not be thought that the problem is confined only to this and similar fields of application of statistical methods. Indeed, it occurs throughout the applied literature (see, e.g., Gower, 2003). Originally, we had thought to address the use of graphics in the whole of applied statistics but it very quickly became apparent that this was too immense a task and we decided to focus on one important field — marketing. There, we found serious deficiencies in labelling diagrams, in identifying the method of analysis used and, above all, in consideration of the shape parameter. As we have seen, getting the shape parameter right is of utmost importance. Primarily, getting the shape parameter right is the responsibility of authors, but editors, providers of software and publishers all have roles to play. If we have done nothing else in this paper, we hope that by drawing attention to this problem things will improve.

We have suggested certain icons to help readers appreciate what can and what cannot be used when interpreting different presentations. We expect that our list of icons can be improved and extended. Just by showing icons, the reader is warned when to be careful interpreting graphical presentations. Perhaps there is more (or maybe less) to maps than meets the eye!

Rather than attempting an exhaustive discussion, we have thought it more important to try and put across our main points. Two important issues that we have omitted, but of which readers should be aware, are (i) the use of calibrated biplot axes and (ii) the concept of linked figures. The former simplify the evaluation of inner-products (see Gower and Hand, 1996). The latter says that if showing two or more related panels in a figure (say dimensions 1 and 2 in one panel and dimensions 3 and 4 in another panel) then it is not sufficient to ensure unit shape factors for each panel, but the length of a unit must also be the same on both panels.

Finally, we mention again that we have been concerned solely with graphical presentation. We have made no attempt to examine whether or not the best or even, appropriate, methods of analysis have been used. That is another story.

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Appendix

A Summary of multidimensional techniques

Table 6 lists some of the more common multidimensional techniques used in marketing, together with the icons appropriate for their interpretation.

A.1 Biplots

The term biplot has been used sporadically throughout the main text. A biplot is a graphical device for showing and associating two kinds of information (the modes) on a single diagram. Typically, we might show samples (cases, units, persons, ...) and variables (responses, scores, category-levels, ...) or the rows and columns of a two-way table. Samples are usually shown as points and numerical variables as vectors though, especially for rows and columns, we may use two types of points (one representing row labels and the other column labels). Distance is the main tool for interpreting differences between pairs of points of like mode. Interpretation of the association between modes may also use distance but it is usually in terms of inner-products, requiring the product of two lengths multiplied by the cosine of their containing angle. This calculation can be simplified by projecting points onto calibrated axes (extended vectors), as for ordinary coordinate axes. Nonlinear axes, and special considerations for categorical variables are available. The ability to associate the representations of the two modes is essential; when unavailable we refer to a joint plot. Biplots of various forms are associated with many multidimensional methods, a few of which are described below.

A.2 Principal Components Analysis (PCA)

The PCA of a data-matrix \mathbf{X} giving measurements on p variables (the columns) and n objects (the rows) has many derivations and interpretations. The simplest, and original is to regard the rows of \mathbf{X} as giving the coordinates of n points in p dimensions and seek an r -dimensional approximation $\hat{\mathbf{X}}$ (typically, $r = 2$). The solution to minimising $\|\mathbf{X} - \hat{\mathbf{X}}\|^2$ is given by the Eckart-Young theorem using the singular value decomposition of $\mathbf{X} = \mathbf{U}_p \mathbf{\Sigma}_p \mathbf{V}'_p$ where $\mathbf{U}'\mathbf{U} = \mathbf{I}$, $\mathbf{V}'\mathbf{V} = \mathbf{V}\mathbf{V}' = \mathbf{I}$ and $\mathbf{\Sigma}$ is diagonal with its (diagonal) elements non-negative and presented in non-increasing order. Then, $\hat{\mathbf{X}} = \mathbf{U}\mathbf{\Sigma}\mathbf{J}\mathbf{V}'$ where \mathbf{J} has its first r diagonal values units, else zero. We may plot the rows of $\mathbf{U}\mathbf{\Sigma}\mathbf{J}$ as n points P_1, P_2, \dots, P_n in r dimensions, giving a representation that may be interpreted in terms of approximations to the distances between the rows of \mathbf{X} . We may also plot $\mathbf{V}\mathbf{J}$ to give points Q_1, Q_2, \dots, Q_p on p axes, the inner product of $\mathbf{U}\mathbf{\Sigma}\mathbf{J}'$ and $\mathbf{V}\mathbf{J}$ giving the approximation $\hat{\mathbf{X}}$ itself. This inner product $(OP_i)(OQ_j) \cos(P_iOQ_j)$

Table 6: Overview of perceptual maps for different techniques and the icons for admissible interpretations. Inadmissible interpretations are implicit as some icons are omitted.

Technique	Plot	Icons		
		Rows or Respondents	Columns or Variables	Rows and Cols or Resp. and Var.
PCA	Biplot: Respondents as points, variables represented as vectors, eigenvalues spread over the variables,			
PCA	Respondents as points, eigenvalues spread over respondents, variables represented as vectors			
CA	Biplot: Rows and columns as points, inertia spread over the rows (row principal standardization, $\alpha = 1, \beta = 0$)			
CA	Biplot: Rows and columns as points, inertia spread over the columns (column principal standardization, $\alpha = 0, \beta = 1$)			
CA	Biplot: Rows and columns as points, inertia spread equally the columns (canonical standardization, $\alpha = \frac{1}{2}, \beta = \frac{1}{2}$)			
CA	Joint plot of rows and columns as points, inertia spread twice, once over rows and again over the columns (symmetric 'French' standardization, $\alpha = 1, \beta = 1$)			
MDS	Plot of the objects			
Unfolding (ideal point model)	Biplot: respondents as points, items as points			
Unfolding (vector model)	Biplot: respondents as vectors, items as points			

may be evaluated directly or by projecting the point P_i onto the biplot axis OQ_j endowed with calibrations.

The vectors \mathbf{V} are often termed the loadings of the derived variables \mathbf{XV} , the so-called principal components. Principal components are often regarded as interpretable latent variables, a process sometimes called reification. Note that $\mathbf{X}(\mathbf{VJ}) = \mathbf{U}\Sigma\mathbf{J}$, the plotted points.

A somewhat different approach is concerned with the approximation of $\mathbf{X}'\mathbf{X}$ (the covariance or, after normalisation, the correlation matrix). Because $\mathbf{X}'\mathbf{X} = \mathbf{V}\Sigma^2\mathbf{V}'$, approximations to the variances/correlations are obtained by plotting $\mathbf{V}\Sigma\mathbf{J}$ and using its innerproduct with itself. The inner product giving $\hat{\mathbf{X}}$ may be completed by plotting \mathbf{UJ} in association with $\mathbf{V}\Sigma\mathbf{J}$. This usage, together with the reification of latent variables is strongly influenced by factor analysis with which PCA is often confused.

A.3 Correspondence Analysis (CA)

In principal, correspondence analysis is concerned with the analysis of contingency tables. Mathematically, however, the method can be applied to any nonnegative data matrix. The objective of correspondence analysis is to give an graphical representation of both rows and columns of the contingency table. For this purpose, high dimensional data is approximated in a low (usually two) dimensional space. There are many ways to describe and define CA. Here, we confine ourselves to a brief summary of the most common configurations obtained by CA. Let \mathbf{X} be the nonnegative data matrix with row and column sums $\mathbf{R1}$, $\mathbf{C1}$ where \mathbf{R} and \mathbf{C} are diagonal and $\mathbf{X1} = \mathbf{R1}$ and $\mathbf{X}'\mathbf{1} = \mathbf{C1}$. Also, $\mathbf{1}'\mathbf{X}\mathbf{1} = \mathbf{1}'\mathbf{R}\mathbf{1} = \mathbf{1}'\mathbf{C}\mathbf{1} = n$ the total of the entries in \mathbf{X} . Visualisations are based on the singular value decomposition $\mathbf{R}^{-1/2}(\mathbf{X} - \frac{1}{n}\mathbf{R}\mathbf{1}\mathbf{1}\mathbf{C}')\mathbf{C}^{-1/2} = \mathbf{U}\Sigma\mathbf{V}'$.

Row and column coordinates are defined as $\mathbf{A} = \mathbf{R}^{-1/2}\mathbf{U}\Sigma^\alpha\mathbf{J}$ and $\mathbf{B} = \mathbf{C}^{-1/2}\mathbf{V}\Sigma^\beta\mathbf{J}$ respectively. In CA, four sets of α and β are commonly distinguished.

1. $\alpha = 1$ and $\beta = 0$. In this setting, the rows of \mathbf{A} are referred to as principal coordinates and the rows of \mathbf{B} give standard coordinates. The corresponding plot is a biplot. Distances between row points are (approximated) chi-squared distances. This setting is also called the row-principal normalisation.
2. $\alpha = 0$ and $\beta = 1$. In this setting, the rows of \mathbf{B} are referred to as principal coordinates and the rows of \mathbf{A} give standard coordinates. The corresponding plot is a biplot. Distances between column points are (approximated) chi-squared distances. This setting is also called the column-principal normalisation.

3. $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{2}$. This setting again yields a biplot. It may be referred to as “symmetrical CA biplot”, but is also known as canonical biplot.
4. $\alpha = 1$ and $\beta = 1$. A joint plot of \mathbf{A} and \mathbf{B} is sometimes referred to as a symmetrical CA plot. However, it should be noted that in this setting we are in fact merging two different plots. Distances between rows are (approximated) chi-squared distances as are the distances between columns. However, distances between row and column points are not defined. Moreover, angles between row and column points cannot be meaningfully interpreted.

For cases 1, 2 and 3 we have that the inner product $\mathbf{AB}' = \mathbf{R}^{-1/2}(\mathbf{U}\Sigma\mathbf{V}')\mathbf{C}^{-1/2} = \mathbf{R}^{-1}(\mathbf{X} - \frac{1}{n}\mathbf{R}\mathbf{1}\mathbf{1}\mathbf{C}')\mathbf{C}^{-1} = \mathbf{R}^{-1}\mathbf{X}\mathbf{C}^{-1} - \frac{1}{n}\mathbf{1}\mathbf{1}'$. The quantity $n\mathbf{R}^{-1}\mathbf{X}\mathbf{C}^{-1}$ is known as the contingency ratio and, assuming independence, gives the ratio *observed/expected* for each cell of \mathbf{X} . This shows that the inner-product projections indicate whether the contingency ratio is greater or less than unity.

It is sometimes said that CA can represent three things (row-distances, column-distances and inner-products) but only two of these may be approximated in any map.

A.4 Multiple Correspondence Analysis (MCA)

When \mathbf{X} is replaced by \mathbf{G} , an indicator matrix for p categorical variables with a single dummy variable for every category of every variable, we may formally analyse \mathbf{G} by CA as if it were a two-way contingency table. Alternatively, as in the PCA of a covariance or correlation matrix, we may approximate $\mathbf{G}'\mathbf{G}$, known as the Burt matrix. Rather than covariances, this gives all the $p(p-1)/2$ two-way contingency tables. Writing $\mathbf{L} = \text{diag}(\mathbf{G}'\mathbf{G})$, the normalised Burt matrix becomes $\mathbf{L}^{-1/2}\mathbf{G}'\mathbf{G}\mathbf{L}^{-1/2}$ which gives the contingency tables in the form $\mathbf{R}^{-1/2}\mathbf{X}\mathbf{C}^{-1/2}$ as for CA itself. Joint Correspondence Analysis (Greenacre, 1988) is concerned with approximating the off-diagonal blocks of the (normalised) Burt matrix, thus excluding the uninteresting block diagonals (i.e. \mathbf{L} or, with normalisation, \mathbf{I}).

Optimal Scores, Homogeneity Analysis, Multiple Correspondence Analysis are equivalent methods concerned with p categorical variables observed on n cases. It is desired to replace the categories by numerical values (the scores) that minimise the variability among the scores within cases, relative to the total variability. Ordinal restrictions may be admitted. Rather similarly Nonlinear PCA, increasingly called categorical PCA, also seeks scores that maximise the PCA fit in some nominated number r of dimensions, especially for $r = 2$.

A.5 Multidimensional Scaling (MDS)

MDS is concerned with drawing a map from a matrix giving the dissimilarities d_{ij} between all pairs of n objects. Thus, the data are like the road distance tables given in road gazetteers. MDS exists in two forms: metric MDS and nonmetric MDS. In metric MDS, the objective is to approximate the given d_{ij} by actual distances between n points in r dimensions (usually $r = 2$). In nonmetric MDS the objective is not for the d_{ij} to approximate the dissimilarities but merely their order. Thus, on the face of things, nonmetric MDS applies to much softer data than does metric MDS but, in practice, there is less difference between maps produced by the two approaches than might be expected.

There are several variants of both metric and nonmetric MDS but all are concerned with optimising some criterion that measures the degree of fit between the given dissimilarities and fitted distances. Some criteria lead to sophisticated algorithms and associated software.

Multidimensional unfolding is a variant of MDS which draws maps given all pairs of distances between p row-objects and q different column-objects. Originally, unfolding was concerned with trying to map in one dimension how p persons scored their perceptions on q objects, in such a way that the points representing objects scored highly by an individual, would lie near the point for that individual. Multidimensional unfolding is the extension to more than one dimension.

Another version of multidimensional unfolding is known as the Vector Model that models the preferences of people for items in a similar biplot representation as in PCA. However, in the vector model of unfolding, the people are represented by vectors and the items by points. A point projecting highly on the direction of a person indicates strong preference of the person for this item. The computations can be done through a standard PCA, usually having the persons as columns and items as rows so that default standardisation in PCA removes the average preference of the people and assumes equal spread of the preferences. The MDPREF model, which is popular in marketing, adds a unit length restriction to the vectors representing the people.